

**AN ELEMENTARY TREATISE
ON THE PLANETARY THEORY:
WITH A COLLECTION OF
PROBLEMS, PP. 1-166**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649506521

An Elementary Treatise on the Planetary Theory: With a Collection of Problems, pp. 1-166 by
C. H. H. Cheyne & A. Freeman

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C. H. H. CHEYNE & A . FREEMAN

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AN
ELEMENTARY TREATISE
ON
THE PLANETARY THEORY,

WITH A COLLECTION OF PROBLEMS.

*Charles
Hartwell*
BY THE LATE
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AND ASSISTANT MATHEMATICAL MASTER IN WESTMINSTER SCHOOL.

THIRD EDITION.

EDITED BY THE
REV. A. FREEMAN, M.A., F.R.S.
RECTOR OF MURSTON, HERTS,
AND LATE FELLOW OF ST JOHN'S COLLEGE, CAMBRIDGE.

London:
MACMILLAN AND CO.
1883

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Cambridge :

PRINTED BY C. J. CLAY, M.A. AND SON,
AT THE UNIVERSITY PRESS.

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Revised 12-9-37 cjm

PREFACE TO THE THIRD EDITION.

At the request of the Rev. Charles Cheyne, the father of the late C. H. H. Cheyne the author of this work, who to the great regret of all his relatives and friends died at Torquay on the 1st of January, 1877, I have undertaken the charge of this new edition. I have made but two changes in the body of the work, and have introduced but few notes. Inasmuch as the methods of Chapter II. are peculiarly the author's own I have not ventured to alter them: but I have instead added to the Appendix the Corollary at the end of Art. 5, a method of determining the variation of the elements (Art. 6) which is for the most part due to Pontécoulant with a slight addition of my own to make it complete, together with Art. 8 also largely due to the same author. I have also given some additional examples and have indicated the sources from which they were obtained.

If a new treatise on the Planetary Theory had to be written it would be necessary to digest the work of Jacobi, Hansen, Leverrier and Newcombe, to mention only a few eminent analysts. A brief account of a variety of methods might be found in Matthieu's *Dynamique*.

A biographical notice of the late author may be read in the *Monthly Notices of the Royal Astronomical Society*, Vol. xxxvii. pp. 147, 148. This I find as complete as is required in such a sketch of the too brief life of a diligent lover of Mathematical and Astronomical Science.

A. FREEMAN.

MURSTON RECTORY, KENT,
October 31st, 1883.

MEMOIRS BY C. H. H. CHEYNE.

1. On the Variation of the Elements in the Planetary Theory. (*Quarterly Journal of Pure and Applied Mathematics*, January, 1861.)
2. On the Variation of the Elements in the Planetary Theory. Second Paper. (*Quarterly Journal of Mathematics*, May, 1861.)
3. On the Variations of the Node and Inclination in the Planetary Theory. (*Quarterly Journal of Mathematics*, October, 1861.)
4. On the Equations of Motion of a Planet referred to Moving Axes. (*Oxford, Cambridge, and Dublin Messenger of Mathematics*, November, 1861.)

PREFACE TO THE FIRST EDITION.

IN this volume, an attempt has been made to produce a Treatise on the Planetary Theory, which, being elementary in character, should be so far complete, as to contain all that is usually required by students in this University. But it is not without diffidence that I submit my volume to their notice. In the earlier part of it, the methods which have been adopted are to some extent original*, and the general arrangement of the second Chapter will, it is believed, be found to be new. Through the kindness of the Publishers, a portion of Pratt's *Mechanical Philosophy* has been placed at my disposal. Of this I have availed myself, particularly in the Chapter on the Stability of the Planetary System; but, on the whole, comparatively little has been reprinted verbatim from that work. Among other sources of information, my obligations are mainly due to Pontécoulant's *Théorie Analytique du Système du Monde*, Airy's *Mathematical Tracts*, and Frost's *Planetary Theory* in the *Quarterly Journal of Mathematics*: but I have also referred to the

* Some of these have already appeared in *Mathematical Journals*.

Mécanique Céleste, the *Mécanique Analytique*, Mrs Somerville's *Mechanism of the Heavens*, (a work forming a complete Mathematical Treatise on Physical Astronomy,) a Memoir by Prof. Donkin on the *Differential Equations of Dynamics*, *Phil. Trans.* 1855, &c. A collection of Problems has been added, taken chiefly from the Smith's Prize and Senate-House Examination Papers of the last twenty years. In conclusion, I would express my sincere thanks to Messrs. A. Freeman, P. T. Main, and other friends, of St John's College, for the valuable assistance which they have afforded me, and would venture to hope that the work will be found useful.

C. H. H. CHEYNE.

ST JOHN'S COLLEGE,
October, 1882.

IN the SECOND EDITION comparatively few changes have been made. The work has been revised, and, it is hoped, in some degree improved. The Stability of the Planetary System has been rather more fully treated, and an elegant geometrical explanation of the formulæ for the secular variations of the node and inclination introduced, for which I am indebted to a paper by Mr H. M. Taylor, Fellow of Trinity College, in the *Oxford, Cambridge and Dublin Messenger of Mathematics*.

C. H. H. C.

1, DEAN'S YARD, WESTMINSTER,
September, 1870.

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