

**ELEMENTS OF  
GEOMETRY AND  
CONIC SECTIONS**

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Elements of geometry and conic sections by Elias Loomis

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**ELIAS LOOMIS**

**ELEMENTS OF  
GEOMETRY AND  
CONIC SECTIONS**



E L E M E N T S

OF

G E O M E T R Y

AND

C O N I C S E C T I O N S.

BY ELIAS LOOMIS, LL.D.,

PROFESSOR OF NATURAL PHILOSOPHY AND ASTRONOMY IN YALE COLLEGE, AND AUTHOR OF  
A "COURSE OF MATHEMATICS."

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TO THE

HON THEODORE FRELINGHUYSEN, LL.D.,

CHANCELLOR OF THE UNIVERSITY OF THE CITY OF NEW YORK.

THE FRIEND OF EDUCATION, THE PATRIOT STATESMAN,

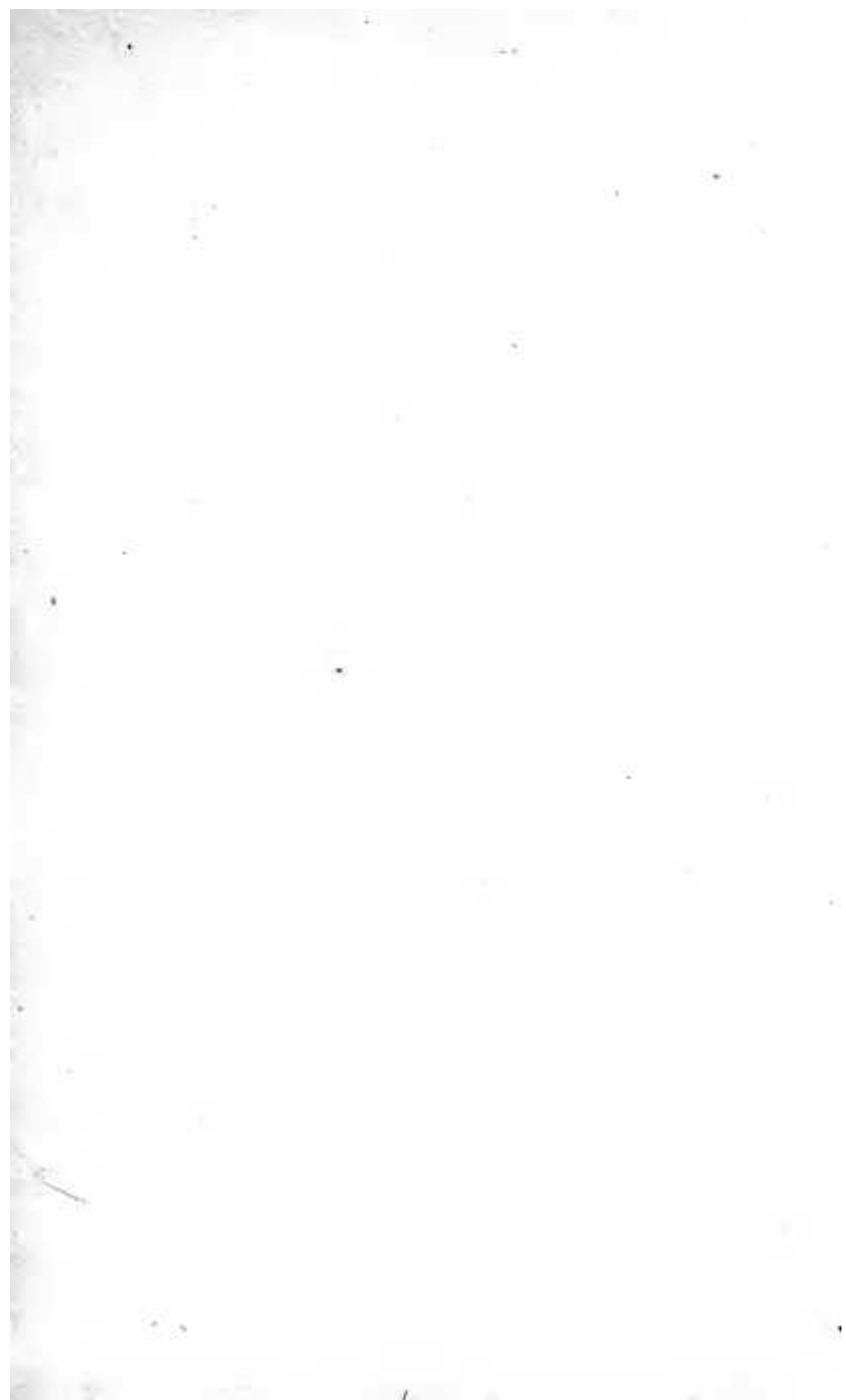
AND THE CHRISTIAN PHILANTHROPIST,

**This Work**

IS RESPECTFULLY DEDICATED

BY

THE AUTHOR.





## P R E F A C E.

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IN the following treatise, an attempt has been made to combine the peculiar excellencies of Euclid and Legendre. The Elements of Euclid have long been celebrated as furnishing the most finished specimens of logic; and on this account they still retain their place in many seminaries of education, notwithstanding the advances which science has made in modern times. But the deficiencies of Euclid, particularly in Solid Geometry, are now so palpable, that few institutions are content with a simple translation from the original Greek. The edition of Euclid chiefly used in this country, is that of Professor Playfair, who has sought, by additions and supplements, to accommodate the Elements of Euclid to the present state of the mathematical sciences. But, even with these additions, the work is incomplete on Solida, and is very deficient on Spherical Geometry. Moreover, the additions are often incongruous with the original text; so that most of those who adhere to the use of Playfair's Euclid, will admit that something is still wanting to a perfect treatise. At most of our colleges, the work of Euclid has been superseded by that of Legendre. It seems superfluous to undertake a defense of Legendre's Geometry, when its merits are so generally appreciated. No one can doubt that, in respect of comprehensiveness and scientific arrangement, it is a great improvement upon the Elements of Euclid. Nevertheless, it should ever be borne in mind that, with most students in our colleges, the ultimate object is not to make profound mathematicians, but to make good reasoners on ordinary subjects. In order to secure this advantage, the learner should be trained, not merely to give the outline of a demonstration, but to state every part of the argument with minuteness and in its natural order. Now, although the model of Legendre is, for the most part, excellent, his demonstrations are often mere skeletons. They contain, indeed, the essential part of an argument; but the general student does not derive from them the highest benefit which may accrue from the study of Geometry as an exercise in reasoning.

While, then, in the following treatise, I have, for the most part, followed the arrangement of Legendre, I have aimed to give his demonstrations somewhat more of the logical method of Euclid. I have also made

some changes in arrangement. Several of Legendre's propositions have been degraded to the rank of corollaries, while some of his corollaries and scholiums have been elevated to the dignity of primary propositions. His lemmas have been proscribed entirely, and most of his scholiums have received the more appropriate title of corollary. The quadrature of the circle is developed in an order somewhat different from any thing I have elsewhere seen. The propositions are all enunciated in general terms, with the utmost brevity which is consistent with clearness; and, in order to remind the student to conclude his recitation with the enunciation of the proposition, the leading words are repeated at the close of each demonstration. As the time given to mathematics in our colleges is limited, and a variety of subjects demand attention, no attempt has been made to render this a *complete* record of all the known propositions of Geometry. On the contrary, nearly every thing has been excluded which is not essential to the student's progress through the subsequent parts of his mathematical course.

Considerable attention has been given to the construction of the diagrams. I have aimed to reduce them all to nearly uniform dimensions, and to make them tolerable approximations to the objects they were designed to represent. I have made free use of dotted lines. Generally, the black lines are used to represent those parts of a figure which are directly involved in the statement of the proposition; while the dotted lines exhibit the parts which are added for the purposes of demonstration. In Solid Geometry the dotted lines commonly denote the parts which would be concealed by an opaque solid; while in a few cases, for peculiar reasons, both of these rules have been departed from. Throughout Solid Geometry the figures have generally been shaded, which, in addition, it is hoped, will obviate some of the difficulties of which students frequently complain.

The short treatise on the Conic Sections appended to this volume is designed particularly for those who have not time or inclination for the study of Analytical Geometry. Some acquaintance with the properties of the Ellipse and Parabola is indispensable as a preparation for the study of Mechanics and Astronomy. Those who pursue the study of Analytical Geometry can omit this treatise on the Conic Sections if it should be thought desirable. It is believed, however, that some knowledge of the properties of these curves, derived from geometrical methods, forms an excellent preparation for the Algebraical and more general processes of Analytical Geometry.

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