

**AN ELEMENTARY TREATISE UPON  
THE METHOD OF LEAST  
SQUARES WITH NUMERICAL  
EXAMPLES OF ITS APPLICATIONS**

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An Elementary Treatise Upon the Method of Least Squares with numerical examples of its applications by George C. Comstock

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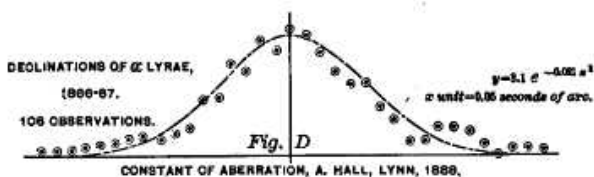
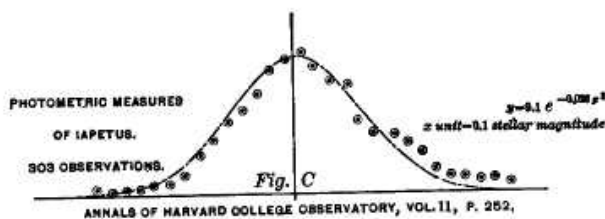
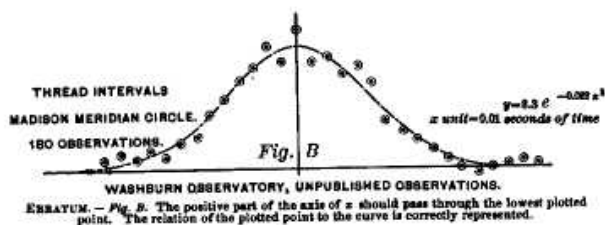
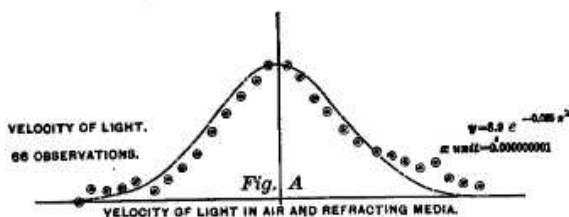
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**GEORGE C. COMSTOCK**

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TYPICAL ERROR CURVES.

AN ELEMENTARY TREATISE

UPON THE

METHOD OF LEAST SQUARES,

WITH

NUMERICAL EXAMPLES OF ITS APPLICATIONS.

BY

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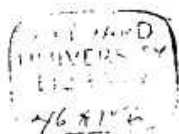
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## PREFACE.

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THE following elementary treatment of the Method of Least Squares has grown out of my attempts to so present the subject to students of physics, astronomy, and engineering, that a working knowledge based upon an appreciation of its principles might be acquired with a moderate expenditure of time and labor.

Conceiving that the ultimate warrant for the legitimacy of the method itself is to be found in the agreement between the observed distribution of residuals and the distribution represented by the error curve, I have not scrupled to abandon altogether the analytical demonstrations of the equation of this curve and to present it as an empirical formula, representing the generalized experience of observers. The evidence in support of a formula of this kind is necessarily cumulative, and the few curves which are presented in illustration of the law of error are to be considered as samples of the kind of evidence which exists in great abundance. By abandoning the theoretical demonstrations, the student is freed from the embarrassments which are usually encountered at the threshold of the subject, and which in many cases cause it to appear as a mathematical puzzle whose analytical difficulties absorb the attention of the tyro to the complete exclusion of the purposes for which the analysis is conducted.

I have sought to give prominence to the distinction between accidental and systematic errors, and to insist upon the limi-



tations which result from the difference between these two classes of error. To illustrate the principles of the text, I have made free use of numerical data and have arranged the computations in forms which experience has shown to be convenient for the purpose, with a view to their subsequent use by the student as models for his own computations.

In the preparation of these pages, I have consulted many, if not most, of the standard treatises upon the subject, but my indebtedness for suggestions and methods of treatment is principally to

FAYE, *Cours d'Astronomie de l'École Polytechnique.*

OPPOLZER, *Lehrbuch der Bahnbestimmung.*

WRIGHT, *Treatise on the Adjustment of Observations.*

G. C. C.

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