ELEMENTS OF THE MATHEMATICAL THEORY OF FLUID MOTION. WAVE AND VORTEX MOTION, PP. 1-177

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649516490

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BY

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REPRINTED FROM VAN NOSTRAND'S MAGAZINE.

NEW YORK:
D. VAN NOSTRAND, PUBLISHER,
23 MURRAY AND 27 WARREN STREET.
1879.

PREFACE.

The subject of Hydrodynamics embraces many of the most difficult problems in the range of physical research. Although, at all times attracting the attention of the greatest minds, it is only within little over a century past that much real progress has been made in the solution of the many and complicated cases presented by the ordinary phenomena of Fluid Motion. The names of Euler, Lagrange and Laplace in the last century, and of Helmholtz, Stokes, Thomson, Rayleigh and Kirchhoff in this, stand out preeminently as those that have done the most to advance the theory to its present position. The object of the following article is to present in a short space the more important points in the Mathematical Theory of Fluid Motion, as it has been developed by these investigators. It is a want severely felt by any one making a study of this subject, that there exists no separate and complete treatise on Hydrodynamics.

It is a fact, I think, greatly to be regretted, that the men who do the most for the real advancement of science so seldom present to the world the result of their labors and extensive knowledge, in any other form than an occasional memoir in a scientific journal, or in a communication to a learned society. There are, however, notable exceptions to this general rule, as witness: Maxwell's treatise on Electricity and Magnetism; Rayleigh on Sound, Cayley's Elliptic Functions, and a few others. If some one would present to the public a treatise on Hydrodynamics, of the scope of those mentioned on other subjects, he would certainly receive the gratitude of all physical students, and confer a great boon upon the scientific world.

T. C.

BALTIMORE, *April*, 1879.

Elements of the Mathematical Theory

OF

FLUID MOTION.

THE following paper contains the mathematical investigation of some of the cases of the motion of incompressible, frictionless fluids. The results obtained are to be considered as applying only to this class of fluids, unless the contrary is expressly stated. The paper is intended to be introductory to a treatise which I hope before long to be able to publish.

In a subject so difficult as Hydrodynamics, there is but little chance for the discovery of hitherto unheard of properties of the quantities dealt with, so, in what follows, the reader will not look for much that is absolutely new in the way of fact, although the arrangement of the work and in many cases the methods employed are my own.

The references to the original sources from which information has been drawn, are given in every case, and I trust that these references, together with the matter contained in this paper, will prove of value to any one interested in the most difficult but beautiful problem of fluid motion.

Of late years, much has appeared in different places upon the subject of Hydrodynamics, but, so far as I am aware, there is no general work either in the English, French or German languages. The aim of this paper and the treatise which will follow will be to combine in one work, all of importance that has been written upon the subject, and so enable the student to forego the immense amount of research necessary in order thoroughly to inform himself upon any one branch of the subject.

The short section which appears upon the theory of the Potential, is principally taken from Clausius's work upon that subject. The references to theoretical mechanics are, unless otherwise stated, to Thomson and Tait's Natural Philosophy. Kirchhoff's Mathematische Physik, and Clifford's Elements of Dynamic, have also been consulted.

§ I.

GENERAL EQUATIONS OF FLUID MOTION.

Let X, Y, Z denote as usual the component forces acting at the point (x, y, z) of the fluid reckoned per unit of its mass —then denoting by ρ the density of the fluid we have for the forces acting upon the elementary mass $\rho dxdydz$ the expressions

$$X\rho dxdydz, \quad Y\rho dxdydz, \quad Z\rho dxdydz;$$

Now for the fluid pressure acting upon one face of the elementary parallelopiped, say, $\delta y\delta z$ we have $p\ \delta y\delta z$, p denoting the pressure on unit of area; upon the opposite face it is, neglecting powers of δx higher than the first,

$$-\delta y\delta z\left(p+\frac{dp}{dx}\delta x\right)$$