ORDINARY DIFFERENTIAL EQUATIONS: AN ELEMENTARY TEXT-BOOK. WITH AN INTRODUCTION TO LIE'S THEORY OF THE GROUP OF ONE PARAMETER

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BY

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PREFACE.

This elementary text-book on Ordinary Differential Equations, is an attempt to present as much of the subject as is necessary for the beginner in Differential Equations, or, perhaps, for the student of Technology who will not make a specialty of pure Mathematics. On account of the elementary character of the book, only the simpler portions of the subject have been touched upon at all; and much care has been taken to make all the developments as clear as possible—every important step being illustrated by easy examples.

In one material respect, this book differs from the older text-books upon the subject in the English language: namely, in the methods employed. Ever since the discovery of the Infinitesimal Calculus, the integration of differential equations has been one of the weightiest problems that have attracted the attention of mathematicians. It is not possible to develop a method of integration for *all* differential equations; but it was found possible to give theories of integration for certain classes of these equations; for instance, for the *homogeneous* or for the *linear*, differential equation of the first order. Also, important theories for the linear differential equations of the second or higher orders, have been developed. But all these special theories of integration were regarded by the older mathematicians as *different* theories based upon separate mathematical methods.

Since the year 1870, Lie has shown that it is possible to subordinate all of these older theories of integration to a general *method*: that is, he showed that the older methods were applicable *only* to such differential equations as admit of known infinitesimal transformations. In this way it became possible to derive all of the older theories from a common source: and at the same time, to develop a wider point of view for the general theory of differential equations.

Only a very small part of Lie's extensive and important developments upon these subjects could, however, be presented in a text-book intended for beginners. The memoirs published by Lie on differential equations are to be found in the "Verhandlungen der Gesellschaft der Wissenschaften zu Christiania," 1870-74; in the *Mathematische Annalen*, Vol. II., 24 and 25; and in his *Vorlesungen über Differentialgleichungen mit Bekannten Infinitesimalen Transformationen*, edited by Dr. G. Scheffers, Teubner, 1891. Besides these sources of information, the writer had the advantage of hearing, in 1886-87, at the same time with Dr. Scheffers, Prof. Lie's first lectures upon these subjects at the University of Leipzig.

All the methods, depending upon the theory of transformation groups, employed in Chapters III.-V., and IX.-XII. of this book, are due *exclusively* to Prof. Lie.

Lie has also developed elegant theories of integration for Clairaut's and Riccati's equations, as well as for the general linear equation with constant coefficients; but, as an exposition of these theories requires a more extensive preparation than it was considered advisable to give in a purely elementary text-book, the author determined to follow, in the treatment of the above-mentioned equations, the older methods—hoping to present Lie's methods for these equations, as well as some of his more far-reaching theories, in a second volume.

In the preparation of this book the author has made free use of the examples in the current English text-books: and he is under special obligations to the works of Boole, Forsyth, Johnson, and Osborne. The treatment of Riccati's equation, Chapter VII., is substantially that given by Boole.

The arrangement of the matter will be found sufficiently indicated by the table of contents; and an index is given at the end of the book.

The articles in the text printed in small type may be omitted by the reader who is going over the subject for the first time.

JAMES MORRIS PAGE.

Johns Hopkins University,
Baltimore, U.S.A.,
July, 1896.

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