

# **ANALYTIC GEOMETRY**

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Analytic Geometry by W. A. Wilson & J. I. Tracey

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**W. A. WILSON & J. I. TRACEY**

**ANALYTIC  
GEOMETRY**



# ANALYTIC GEOMETRY

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## PREFACE

THE purpose of this book is to present in a short course those parts of Analytic Geometry which are essential for the study of Calculus. The material has been so arranged that topics which are less important may be omitted without a loss of continuity, and the text is therefore adapted for use in classes which aim to cover in one year the fundamental principles and applications of both Analytic Geometry and Calculus. For such classes Chapters I-VI, VIII, and the earlier sections of Chapters VII and IX form a course which can be completed in about thirty-five lessons. In order to provide material for study if more time is available, a discussion of the general equation of the second degree and short chapters on Tangents and Normals and Solid Analytic Geometry have been included.

In the preparation of the book the authors have used as a foundation a pamphlet written by Professor William Beebe for use in his classes. The authors wish to express their thanks to Dr. David D. Leib and to Dr. Levi L. Conant for many valuable criticisms and suggestions.

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