

**A COURSE IN MATHEMATICAL
ANALYSIS: FUNCTIONS
OF A COMPLEX VARIABLE,
BEING PART I OF VOLUME II**

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A Course in Mathematical Analysis: Functions of a Complex Variable, Being Part I of Volume II by Édouard Goursat & Earle Raymond Hedrick & Otto Dunkel

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ÉDOUARD GOURSAT & EARLE RAYMOND HEDRICK & OTTO DUNKEL

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H. O. Brown

A COURSE IN MATHEMATICAL ANALYSIS

FUNCTIONS OF
A COMPLEX VARIABLE

BEING PART I OF VOLUME II

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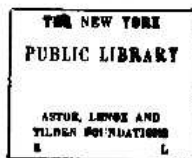
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AUTHOR'S PREFACE—SECOND FRENCH EDITION

The first part of this volume has undergone only slight changes, while the rather important modifications that have been made appear only in the last chapters.

In the first edition I was able to devote but a few pages to partial differential equations of the second order and to the calculus of variations. In order to present in a less summary manner such broad subjects, I have concluded to defer them to a third volume, which will contain also a sketch of the recent theory of integral equations. The suppression of the last chapter has enabled me to make some additions, of which the most important relate to linear differential equations and to partial differential equations of the first order.

E. GOURSAT

Warren and Sons, 9, 2, 54



TRANSLATORS' PREFACE

As the title indicates, the present volume is a translation of the first half of the second volume of Goursat's "Cours d'Analyse." The decision to publish the translation in two parts is due to the evident adaptation of these two portions to the introductory courses in American colleges and universities in the theory of functions and in differential equations, respectively.

After the cordial reception given to the translation of Goursat's first volume, the continuation was assured. That it has been delayed so long was due, in the first instance, to our desire to await the appearance of the second edition of the second volume in French. The advantage in doing so will be obvious to those who have observed the radical changes made in the second (French) edition of the second volume. Volume I was not altered so radically, so that the present English translation of that volume may be used conveniently as a companion to this; but references are given here to both editions of the first volume, to avoid any possible difficulty in this connection.

Our thanks are due to Professor Goursat, who has kindly given us his permission to make this translation, and has approved of the plan of publication in two parts. He has also seen all proofs in English and has approved a few minor alterations made in translation as well as the translators' notes. The responsibility for the latter rests, however, with the translators.

E. R. HEDRICK
OTTO DUNKEL

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