

**EASY RULES FOR THE
MEASUREMENT OF
EARTHWORKS: BY MEANS OF
THE PRISMOIDAL FORMULA**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649567478

Easy Rules for the Measurement of Earthworks: By Means of the Prismoidal Formula by
Ellwood Morris

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ELLWOOD MORRIS

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EASY RULES

FOR THE

MEASUREMENT OF EARTHWORKS,

BY MEANS OF THE

PRISMOIDAL FORMULA.

ILLUSTRATED WITH NUMEROUS WOODCUTS, PROBLEMS, AND EX-
AMPLES, AND CONCLUDED BY AN EXTENSIVE TABLE
FOR FINDING THE SOLIDITY IN CUBIC
YARDS FROM MEAN AREAS.

THE WHOLE

BEING ADAPTED FOR CONVENIENT USE BY ENGINEERS, SURVEYORS,
CONTRACTORS, AND OTHERS NEEDING CORRECT
MEASUREMENTS OF EARTHWORK.

BY

ELLWOOD MORRIS, CIVIL ENGINEER.

PHILADELPHIA:

T. R. CALLENDER & CO., THIRD AND WALNUT STS.

LONDON: TRÜBNER & CO., 69 PATERNOSTER ROW.

1872.

Dedication.

—
RESPECTFULLY DEDICATED

TO THE

ENGINEERS, SURVEYORS, AND CONTRACTORS

OF

THE UNITED STATES,

BY ONE WHO IS WELL ACQUAINTED

WITH

THEIR ABILITIES AND WORTH.

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CHAPTER I

PRELIMINARY PROBLEMS.

1. *Of the Prismoid.*—Although this solid probably originated with the ancient geometers—THOMAS SIMPSON (1750), an eminent mathematician of the last century, appears to have been *the first*, in later days, to demonstrate the rule for its solidity,* now accepted by modern mensurators; and he was soon followed by Hutton, in his quarto treatise on Mensuration,† who by another process again demonstrated the Prismoidal Rule, and at the same time laid the foundations of modern mensuration, in a manner so solid, that it has come down to our time, through various editors and commentators, *substantially* (in many cases literally) *the same* as established by Hutton in his famous work of 1770.

Simpson's rule for the prismoid has been variously transformed, and written, and is now generally known by the name of *the prismoidal formula*, of which we will give hereafter the usual expressions, as well as some useful modifications, the same in substance, but often more convenient for practical purposes.

The solid called a Prismoid (from its general resemblance to a prism, and in like manner named from its base, triangular, rectangular, trapezoidal, etc.) *is a body contained between two parallel planes,*

* Simpson's Doctrine of Fluxions. (1750), 8vo, London.

† Hutton's Mensuration. (1770), 4to, Newcastle upon Tyne.

its height being their perpendicular distance apart, its ends rectangles, and its faces plane trapezoids;—and this seems to be a sufficient definition. As to such form, all prismoids may be reduced or made equivalent; but although this simple definition answers our purpose of introducing the rectangular prismoid, HUTTON'S, Art. 3, is the authoritative one.

This solid is usually the frustum of a wedge; but, as the proportions of the ends are changed, it may become a frustum of a pyramid, a complete pyramid, a wedge, or a prism; and hence it is indispensably necessary that the rule for its solidity should also hold for all these solids, which, in fact, it does.

The ends may be, and often are, irregular polygons, but they must always coincide with the limiting parallel planes; and though the solid may be quite oblique, its height must be taken normal to the end planes. The faces are usually straight longitudinally, but this condition is not absolute, since the remarkable formula; deduced from the prismoid for its solidity, applies as well to the volume of many curved solids in an extraordinary manner, of which the limits are not yet known, though more than a century has elapsed since Simpson developed it.

The mid-section, included by the usual prismoidal formula, must be in a plane parallel to, and equally distant from, those containing the ends, and is deduced from the arithmetical average of like parts in them. It is entirely hypothetical, or assumed for the purposes of computation, and has no actual existence in the body itself.

The rectangular prismoid (usually regarded as the elementary figure of this solid) is a frustum of the wedge.

(a.) Thus the prismoid AB (Fig. 1) is a frustum of the wedge AEC.

