# SOLUTIONS TO PROBLEMS CONTAINED IN A GEOMETRICAL TREATISE ON CONIC SECTIONS

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649324477

Solutions to Problems Contained in A Geometrical Treatise on Conic Sections by W. H. Drew



Edited by Trieste Publishing Pty Ltd.
Cover @ 2017



www.triestepublishing.com

W. H. DREW

# SOLUTIONS TO PROBLEMS CONTAINED IN A GEOMETRICAL TREATISE ON CONIC SECTIONS

Trieste

# SOLUTIONS TO PROBLEMS

CONTAINED IN

# A GEOMETRICAL TREATISE ON CONIC SECTIONS.

BY THE
REV. W. H. DREW, M.A.
OF ST. JOHN'S COLLEGE, CAMBRIDGE;
SECOND MASTER OF BLACKHEATH PROPRIETARY SCHOOL.

*SECOND EDITION.*

London:
MACMILLAN AND CO.
1868.

# PREFACE.

The present work is designed to be a continuation of, and is intended to be used by the learner in connexion with, my treatise on "Geometrical Conic Sections." The object is not so much to furnish a key, as to illustrate the application of the principles of Geometry to the solution of questions in Conic Sections. It has been too much the custom to despise the use of geometrical methods, and to reject them as uncertain except in the solution of very elementary problems. The present work will, I hope, show that Geometry is a more effective and practical instrument in the treatment of this subject than is generally supposed.

Another object that I have likewise kept in view is to supply a connecting link by which the student may be led from the simple process of following the steps of a demonstration, to devising for himself the solution of an original geometrical problem. Most persons engaged in teaching must have felt the want of some intervening step to bridge over this difficulty. With this purpose the Solutions are all based on one or other of the propositions in the former treatise, and the student is left to modify the construction and figure according to the particular question under consideration. An amount of care and attention will thus be rendered necessary, which cannot fail to insure the complete understanding of the principles of the subject, and which will make it impossible for the learner to acquire an apparent but treacherous knowledge, by the undue use of a retentive memory. Prob. 31 of the Parabola, Prob. 55 of the Ellipse, and Prob. 32 of the Hyperbola, afford good illustrations of the plan I have pursued, which really indicates the way in which the properties were themselves suggested.

It has been found convenient to refer occasionally to a *new edition* of the "Conic Sections," which is now in course of publication. In this edition such slight modifications, as experience has shown to be needed, will be introduced. A considerable number of *unsolved* problems will be added, and the work will be brought completely up to the requirements of the present time.

I am indebted for much valuable help in the preparation of these "Solutions," to Mr. A. Freeman, of St. John's College, and to Mr. F. R. Drew, of Sidney College.

W. H. DREW.

BLACKHEATH PROPRIETARY SCHOOL,
*November 20th*, 1861.

# CONIC SECTIONS.

## PROBLEMS ON THE PARABOLA.

1. Draw $BQ$ at right angles to $AB$; then

$$AS \cdot SQ = BS^2 = 4AS^2,$$
$$\therefore SQ = 4AS, \text{ and } AQ = 5AS.$$

2. The triangles $PNG$, $GPK$ are equiangular, and have the side $PG$ common,

$$\therefore \text{ they are equal in all respects,}$$
$$\therefore PK = NG = 2AS.$$

3. $SP = SG = 2NG = 4AS = BC.$

4. Produce $PQ$ to meet the axis in $T$, and join $AQ$; then,

$$ST \cdot SA = SQ^2 = 4AS^2,$$
$$\therefore ST = 4AS, \text{ and } SN = 2AS,$$
$$\therefore SA : SQ :: SN : SP,$$
$$\therefore \angle SQA = \angle SPN, \text{ or } \angle QSB = \angle PSB.$$

5. Since $PSZ$ is a right angle, and $SY$ perpendicular to $PZ$,

$$\because PY \cdot PZ = SP^2,$$
$$\text{also } PY \cdot YZ = SY^2 = SA \cdot SP.$$

6. $$AN \cdot NL = PN^2 = 4AS \cdot AN,$$
$$\therefore NL = 4AS.$$

7. Let the tangent at $P$ meet the latus rectum in $L$; then

$$ZY : YL :: XA : AS,$$

$$\therefore\ ZY = YL,$$

and $SY$ is perpendicular to $ZL$,

$$\therefore\ SZ = SL.$$

8. Since $AT = AN$, and the angles at $A$ are right angles,

$$\therefore\ TY = NY.$$

Also, since the angles at $N$ and $Y$ are right angles, a circle may be described about $SNPY$,

$$\therefore\ TP\,.\,TY = TN\,.\,TS.$$

9. Let the tangent at $P$ meet the tangent at $A$ in $Y$; then since $SYP$ is a right angle, the circle described about $SNP$ will pass through $Y$.

Also, since $AY^2 = AS\,.\,AT = AS\,.\,AN,$

$\therefore\ AY$ is a tangent to the circle.

Now $AY : PN :: AT : TN,$

$$\therefore\ AY = \tfrac{1}{2}PN.$$

10. Let the tangent, ordinate, and normal at $P'$ meet the axis in $T'$, $N'$ and $G'$; then

since $AN' = AT'$ and $N'N = NG'$,

$$\therefore\ AN = \tfrac{1}{2}\,T'G'.$$

Now $PN^2 = 2AS\,.\,2AN = N'G'\,.\,T'G' = P'G'^2,$

$$\therefore\ PN = P'G'.$$

11. From any point $R$ on the tangent at $P$ draw the tangent $RQ$; then the triangles $SQR$, $SPR$ are similar,

$\therefore$ the angle $SRQ =$ the angle $SPR$,

$\therefore$ the $\angle\, SRQ$ is independent of the point $R$.

12. Draw $NQ$ a tangent to the circle.

Now $RN^2 : A'B^2 :: AN^2 : AA'^2,$

and $PN^2 : A'B^2 :: AN : AA',$

$:: AN\,.\,AA' : AA'^2,$

$\therefore\ RN^2 - PN^2 : A'B^2 :: AN\,.\,A'N : AA'^2,$

or $RP\,.\,RP' : NQ^2 :: A'B^2 : AA'^2.$

13. Let $B$ be the point where the axis of the parabola is to touch the circle, and $C$ the other given point.

Draw the tangents $TC$, $TBG$ intersecting in $T$; also draw $CN$ perpendicular to $TG$, and $CG$ passing through the centre $O$ of the circle; then

$NT$ and $NG$ will be respectively the subtangent and subnormal of the parabola required.

Bisect $NT$ in $A$, and $TG$ in $S$; then $A$ and $S$ will be respectively the vertex and focus of the parabola.

14. Let $QT$ be the tangent at the point $Q$.

Through $Q$ draw the chord $QOQ'$; and let $WRO$ be drawn parallel to the axis meeting $QT$ in $W$, the parabola in $R$, and the chord $QQ'$ in $O$.

Bisect $QQ'$ in $V$, and draw the diameter $TPV$ parallel to the axis, and join $SP$.

Now from similar triangles

$$QO : OW :: QV : VT,$$
$$:: QV^2 : QV \cdot VT,$$
$$:: 4SP \cdot PV : 2QV \cdot PV,$$
since $QV^2 = 4SP \cdot PV$ (*Prop.* XV.)
and $VT = 2PV$ (*Prop.* XIV.)
$$\therefore QO : OW :: 4SP : 2QV,$$
$$:: 4SP : QQ',$$
$$\therefore QO \cdot QQ' = 4SP \cdot OW.$$
But $QO \cdot OQ' = 4SP \cdot RO$ (*Prop.* XVII.),
$$\therefore QQ' : OQ' :: OW : RO,$$
$$\therefore QO : OQ' :: WR : RO.$$

15. Join $SP$; then

$$PV = ST = SP = PO.$$

16. Draw $VM$ at right angles to the axis; then

$$\text{since } VM : MS :: PN : NT,$$
$$\therefore MS = NT.$$
$$\text{But } PN^2 = 2\,AS \,.\, NT,$$
$$\therefore VM^2 = 2\,AS \,.\, SM,$$

$\therefore$ the locus of $V$ is a parabola, whose vertex is $S$, and latus rectum half that of the given parabola.

17. Produce $QV$, $UR$ to meet the tangent at $P$ in $W$ and $X$; then as in *Prop.* XIX. it can be proved that $WV$ and $XU$ are each equal to $4\,SP$,

$\therefore$ $WV$ and $XU$ are equal and parallel,

$\therefore$ $UV$ is parallel to the tangent at $P$.

18. By similar triangles

$$CR : CA :: AN : PN,$$
$$\therefore PN \,.\, CR = AC \,.\, AN,$$
$$\text{or } PN^2 = AC \,.\, AN,$$

$\therefore$ the locus of $P$ is a parabola whose axis is $AB$, and latus rectum equal to the given distance $AC$.

19. Let $CP$ and $AR$ intersect in $V$; and draw $VN \perp^r$ to $AC$, and $VM \perp^r$ to the tangent at $A$; then

$$VN : AN :: CR : CA,$$
$$:: CR : CP,$$
$$:: VN : CV,$$
$$\therefore CV = AN = VM,$$

$\therefore$ the locus of $V$ is a parabola whose focus is $C$, and directrix $AM$.

20. Let $OQ$, $OQ'$ be two equal tangents drawn from the point $O$ in the axis; and let them be cut by the tangent $RPR'$ in $R$ and $R'$.