

# **SOLID GEOMETRY**

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Solid Geometry by Mabel Sykes & Clarence E. Comstock

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**MABEL SYKES & CLARENCE E. COMSTOCK**

**SOLID  
GEOMETRY**



UNIV OF CALIFORNIA

# SOLID GEOMETRY

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## THE CONTENTS

|  | PAGE |
|--|------|
| <i>The Preface</i> . . . . .                                       | vii  |
| <b>CHAPTER I. Lines and Planes</b>                                 |      |
| Introductory . . . . .   | 1    |
| Parallel Lines and Planes . . . . .                                | 6    |
| Perpendicular Lines and Planes . . . . .                           | 12   |
| Equal Angles in Space . . . . .                                    | 17   |
| Perpendiculars and Parallels . . . . .                             | 18   |
| Dihedral Angles . . . . .  | 21   |
| Perpendicular Planes . . . . .                                     | 23   |
| Proportional Segments . . . . .                                    | 29   |
| Loci in Space . . . . .  | 29   |
| Projection . . . . .   | 35   |
| Polyhedral Angles . . . . .  | 38   |
| Summary and Supplementary Exercises . . . . .                      | 42   |
| <b>CHAPTER II. Properties of Polyhedrons, Cylinders, and Cones</b> |      |
| Solids in General . . . . .  | 48   |
| Some Elementary Surfaces . . . . .                                 | 48   |
| Prisms . . . . .   | 51   |
| Cylinders . . . . .  | 55   |
| Pyramids . . . . .   | 60   |
| Cones . . . . .  | 64   |
| Regular Polyhedrons . . . . .                                      | 69   |
| Supplementary Exercises . . . . .                                  | 72   |
| <b>CHAPTER III. The Sphere</b>                                     |      |
| Introductory . . . . .   | 76   |
| Tangents to Spheres . . . . .                                      | 77   |
| Determination of Spheres . . . . .                                 | 79   |
| Circles of Spheres . . . . .                                       | 81   |
| Spherical Angles . . . . .   | 86   |
| Spherical Triangles . . . . .                                      | 87   |
| Supplementary Exercises . . . . .                                  | 102  |

## THE CONTENTS

## CHAPTER IV. Areas and Volumes

|  | PAGE |
|--|------|
| Areas of Polyhedrons . . . . .                       | 106  |
| Volumes of Polyhedrons . . . . .                     | 109  |
| The Measurement of Round Bodies in General . . . . . | 125  |
| The Measurement of the Cylinder . . . . .            | 125  |
| The Measurement of the Cone . . . . .                | 129  |
| The Measurement of Frustums of Cones . . . . .       | 132  |
| General Formula . . . . .                            | 134  |
| Volumes by Cavalieri's Theorem . . . . .             | 138  |
| Spherical Measurements . . . . .                     | 142  |
| Summary and Supplementary Exercises . . . . .        | 156  |

## CHAPTER V. Similarity and Symmetry

|                                   |     |
|-----------------------------------|-----|
| Similarity . . . . .              | 162 |
| Symmetry . . . . .                | 172 |
| Miscellaneous Exercises . . . . . | 173 |

---

|  |     |
|--|-----|
| <i>Notes on Arithmetic and Algebra</i> . . . . .             | 174 |
| <i>Tables</i> . . . . .                                      | 180 |
| <i>Outline of Plane Geometry</i> . . . . .                   | 185 |
| <i>References and Topics for Mathematics Clubs</i> . . . . . | 202 |
| <i>The Index</i> . . . . .                                   | 209 |



## THE PREFACE

The *Solid Geometry* is prepared for the same purpose and with the same general features as is the *Plane Geometry*. In both books the two main characteristics are *analysis* and *emphasis*.

One of the great objectives in education is to train young people to attack difficulties through an analysis of the problems presented. It is because of this fact that both the *Plane* and the *Solid Geometry* are prepared as suggestive method texts with the suggestions in the form of a logical analysis.

Moreover, if the mind is to retain any lasting impression of the work covered, distinctions in emphasis are necessary. The material presented in both the *Plane* and the *Solid Geometry* has been arranged with this fact in mind. Attention is called particularly to chapters ii, iv, and v. Chapter ii discusses the nature and properties of the various surfaces and solids ordinarily studied in solid geometry with the exception of the sphere, which is studied in chapter iii. All areas and volumes are considered in chapter iv. Similarity is considered in chapter v.

There are several advantages gained from this arrangement. It enables the pupil to take up the study of areas and volumes with a clear idea of the solids considered. It makes possible a more logical arrangement of material. Cylinders are compared with prisms and cones with pyramids when the properties of these solids are studied; but as the volume of the pyramid is obtained from that of the prism, a different order is used when volumes are studied. Moreover, the theorems concerning areas and volumes may be worked into a logical whole when considered together, which is not possible in the traditional arrangement. For example, Theorems 121, and 122 serve not only as a necessary preparation for

the measurement of the sphere, but also as a fitting climax to the work on cylinders and cones.

Both the *Plane* and the *Solid Geometry* are written with the firm conviction that if geometry is taught by analysis and if at the same time proper distinctions in emphasis are made, pupils will reach the end of their course with more real education and with a much clearer and more lasting impression of the meaning of the great concepts of geometry than can possibly be the case under traditional methods.

Attention is called also to certain minor features:

The approach to the early theorems through the introductory material is natural and easy.

As the first difficulty, perhaps the only real difficulty, in solid geometry, is the inability of pupils to visualize figures in space, the use of models made by pupils is strongly recommended. These models should be used for demonstration work and should precede the use of blackboard figures until the pupil is clearly out of "flatland." If possible, spherical triangles should be studied from a slated globe.

The treatment of loci in §§ 41-47 and the treatment of similarity in chapter v deserve attention.

The formal study of the theory of limits is omitted. The treatment of areas and volumes of round bodies by this method is given in outline only. It is intended merely to make the results appear reasonable to the pupil.

By proper choice of material the study of volumes may be made to depend upon Cavalieri's theorem. See §§ 173-179 and § 154.

The "Notes on Arithmetic and Algebra" and the "Outline of Plane Geometry" given will be found convenient for reference. The "Topics and References for Mathematic Clubs" are intended as suggestions only.

CHICAGO, ILLINOIS  
March, 1922

M. S.  
C. E. C.

# SOLID GEOMETRY

## CHAPTER I

### LINES AND PLANES

#### INTRODUCTORY

##### SUBJECT MATTER OF GEOMETRY

1. In plane geometry our study was confined to figures that could be drawn on a plane and drawn with ruler and compass only. Such figures are constructed of points and lines.

In solid geometry we extend our study to figures in space. These figures are constructed of points, lines, surfaces, and solids.

2. The space occupied by a ball is a geometrical solid. The outside of the ball is a surface; it is the boundary of the solid and separates the space within from the space without (Fig. 1).

In general we may say that—

The space occupied by any object is called a **geometrical solid**.

The boundaries of a solid are called surfaces.

The surfaces of a solid separate the solid from the remainder of space.

We may define a **surface** as any boundary between two parts of space.

The space occupied by a tomato can is a geometrical solid. The surface of the can separates the space within from the space without. We may consider the total surface of the can as composed of three parts separated by the edges of the can (Fig. 2).



FIG. 1



FIG. 2