A TREATISE ON FACTORIAL ANALYSIS, WITH THE SUMMATION OF SERIES; CONTAINING VARIOUS NEW DEVELOPMENTS OF FUNCTIONS, &C.

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INTRODUCTION.

THE greater part of this Work was written some years ago, and certain portions have been recently read before the Royal Society, for which the Author has received the thanks of that learned body. Series enter, more or less, into all our branches of analysis, as well as into many of the higher departments of physical science: such a subject, therefore, viewed in all its aspects and bearings, would require volumes for its adequate discussion. The ablest treatise we have, upon the summation of algebraic series, is, doubtless, Herschel's Examples of Finite Differences; after the profound and original views of this great mathematician, any attempt at further generalization would seem to be almost a fruitless task. The summation of series, by the method of definite integrals, first employed by Euler, has been extended by Poisson, W. S. B. Woolhouse, and other writers; upon this class of series I have not touched, being desirous of maintaining a sort of unity in this little work.

The publication of any thing in mathematics, which professes to be new, is almost sure to excite, in a certain class of men, the sneer of contempt, or that cold expression of heartless approbation, which, as it stops all inquiry, not unfrequently consigns to unmerited oblivion, ideas, which might have become the germ of great discoveries. Has mathematics attained its final development, that it should be deemed impossible for the humble labourer in the field of science to pick up some gleanings of truth? However, it must be stated, that at a period when there is such a vast accumulation of mathematical truth, it would be presumption in any one to lay claim to strict originality; all, therefore, that the Author of the following Treatise can say is, that so far as he knows, the theorems are either new, or obtained by processes which are new. In particular, he would call attention to Theorems 35, 38, 40, 41, 42, 43, 45, 48, 49, 50, 51, 52, 56, 34, and the leading parts of the third and fourth sections. The equality expressed in 43, is an extension of Murphy's Theorem; and 35, is a remarkable generalization of a series due to Poisson.* The symbol, [ ], of a factorial was, the author believes, first employed by Vandermonde, but by the addition of an accent it is, in the following Treatise, rendered more general in its signification, thereby enabling us to express the factorial, as well in an ascending, as in a descending form. Regarded in this light, this symbol is a more general form of a binomial with an exponent; for when the increment is taken zero, the factorial exponent becomes the ordinary one. The symbol, S, of summation, is employed, not only to indicate the sum of a series, but also to abbreviate the form, and point out the law of any complex expression. The connection between this symbol, and that of definite integration, is rendered apparent by Theorem 53.

The subject of series is one of great interest, and, independently of its utility as a branch of mathematics, is highly calculated to enlarge the understanding. It is easy, in many cases, to see how the sum of an infinite series of terms should be equal to a finite magnitude: thus, let a square be drawn to represent a unit, and a line dividing the square into two equal parts or halves; let one of these halves be divided into

* See De Morgan's great work on the Calculus, p. 244.

two equal parts, forming fourths, then one of these fourths into eighths, and so on without end, then it will be apparent that the whole square, or unit, is made up of the terms, one-half, one-fourth, one-eighth, and so on to infinity. Nor are our conceptions of infinity limited to mathematical quantities: some of our sublimest ideas involve the consideration of a series: thus, our conception of Deity is the summation of all that is vast, lovely, beneficent, and holy in the universe,—the function of which peopled immensity is the development,—the last incomprehensible link in the interminable chain of causation.

To those who are in the habit of considering differentiation as the universal solvent of all mathematical difficulties, not a few of the methods of investigation employed in this Work will appear tedious and operose. Taking for granted some of our great analytical principles and theorems, it becomes an easy task to establish, by a deductive process, formulæ of less generality. There is a charm in this deductive process, and a factitious, though an entire, confidence in the accuracy of our results; for we employ formulæ which have received the tribute of admiration from the genius of all nations, and which may have shed a light upon the works of nature, or given rise to some of the most magnificent discoveries in art. It is nevertheless true, that the too frequent use of these general forms, leads us to undervalue principles which would enable us to work out results, more simply, if not more elegantly, than by differentiation or integration. Such methods disguise the real difficulties of a problem; for while it is easy to understand the meaning and use of a general formula, every mind cannot, at once, appreciate the principles of reasoning upon which the demonstration of that formula is based. By the application of a rule, transformations are effected, which save the exercise of thought, and the labour of development; but the rigid solution of a problem, or the complete demonstration of a theorem, calls into vigorous operation the slumbering energies of the higher intellect, and may thus exercise an important influence upon the whole of the student's career. The love of truth is a lamp to the soul, and sheds a holy light over the whole of our destiny; but this sentiment can only be duly cherished by habituating ourselves to exact methods of thought.

Pure algebra has been, for many years, an unproductive, because a comparatively neglected, field of inquiry › proud of the majesty of science, men of great genius have generally lived in a region of transcendentalism and abstraction, adding truth to truth, without deigning to bestow a thought upon humbler, though not less useful, walks of knowledge. So long as the student is confined to the relations of definite magnitudes, his path is easy and pleasant,—he can test the accuracy of his conclusions by an appeal to figures,—he can retrace the steps of his investigations, by new assumptions; but the moment he comes to consider the properties of quantities indefinitely extended or diminished, he passes into a field beyond the ordinary range of his associations, and then the inquiry appears to him wrapped up in all the mystery in which ideas of infinity are commonly involved. The difficulty, however, lies not in the mathematics, but in the metaphysics of the subject; for, viewed as a mathematical fact, nothing can be more simple than the doctrine of ultimate ratios, which arises out of the most simple laws of quantity; and if the student were led to deduce results, by the application of the principle, before any veil is thrown over the process by the hasty adoption of an arbitrary notation, the difficulty, which most students experience, in learning the differential calculus, would be to a great extent obviated. As great principles are rarely, if ever, at first discovered in their most abstract form, so certain initiatory steps of instruction seem best calculated to convey to the mind of a student the knowledge of such abstract truths. In an age, therefore, when not only truth, but the processes by which truth has been attained, are subjects of inquiry, any attempt calculated to enforce the consideration of the more subtile principles of reasoning, ought to meet with a due amount of encouragement.

The fact, that number may be made up in different ways, is the great principle upon which all analytical truths are founded: our most splendid theorems are, therefore, little more than disguised truisms or identities, where the difference of form is owing either to the manner of grouping the elements, or to the circumstance of certain expressions being left in an undeveloped state; but, if every operation be performed, and the terms, on each side of the theorem, arranged according to the powers of any one of the elements, then we shall have, not only an equality of value, but an actual identity of form. As the element of light, modified by the surfaces upon which it falls, gives that endless variety of shade and colour to the objects of creation; so the simple principle referred to, gives to mathematical quantities an infinite variety of forms and modes of development. Mathematical propositions, therefore, are necessary truths; that is, they would remain true, even though there were no intelligencies in the universe to appreciate them: hence we cannot wonder at the certainty which attends such inquiries: and hence too the tendency of minds, exclusively mathematical, to trace everything in nature to the operation of abstract laws, whilst minds of even an inferior order, engaged in the contemplation of those laws and relations of matter which are discoverable by experiment and observation, are led to the firm conviction, that the universe is not like the development of a mathematical quantity, which could not have been otherwise than it