ELEMENTS OF THE THEORY OF THE NEWTONIAN POTENTIAL FUNCTION

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649572472

Elements of the Theory of the Newtonian Potential Function by B. O. Peirce

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

B. O. PEIRCE

ELEMENTS OF THE THEORY OF THE NEWTONIAN POTENTIAL FUNCTION

Trieste

ELEMENTS

OF THE

THEORY OF THE NEWTONIAN POTENTIAL FUNCTION.

BY

B. O. PEIRCE, PH.D.,

ASSISTANT PROFESSOR OF MATHEMATICS AND PHYSICS
IN HARVARD UNIVERSITY.

BOSTON:
PUBLISHED BY GINN & COMPANY.
1886.

J. S. Cushing & Co., Printers, Boston.

PREFACE.

THIS book is almost entirely made up of lecture-notes which from time to time during the last four years I have written out for the use of students who have begun with me the study of what I have ventured to call, after Neumann, the Newtonian Potential Function.

The notes were intended for readers somewhat familiar with the principles of the Differential and Integral Calculus, but unacquainted with many of the methods commonly used in applying Mathematics to the study of physical problems. These students, I learned, found it difficult to get from any single book in English a treatment of the subject at once elementary enough to be within their easy comprehension, and at the same time suited to the purposes of such of them as intended eventually to pursue the subject farther, or wished, without necessarily making a specialty of Mathematical Physics, to prepare themselves to study Experimental Physics thoroughly and understandingly. What is here printed seems to have been of use to some of those who have read it in manuscript, and it is hoped that it may now be helpful to a larger number of students.

Since these notes are professedly elementary in character, I feel that no apology is needed for what may seem to be the rather prolix way in which some of the subjects are treated, or for an arrangement of matter which would be

unsuitable in a book intended for a different class of readers. I have not hesitated to use a long proof whenever this has seemed to me more easily comprehensible than a short and mathematically neater one, and I have often given more than one demonstration of a single theorem in order to illustrate different methods of work. Although I have used freely the notation* of the Calculus, I have assumed on the part of the reader only an elementary knowledge of its principles.

The short treatment of Electrostatics in Chapter v. is introduced to show how the theorems of the preceding chapters may be used in solving physical problems; but it is hoped that a person who has mastered even the little here given will be able to understand, with the aid of some good treatise on Experimental Physics, most of the phenomena of Electrostatics. It is also hoped that those readers who mean to study the subject of Electricity from the mathematical point of

* In this book the change made in a function u by giving to the independent variable x the arbitrary, finite increment Δx, and keeping the other independent variables, if there are any, unchanged, is denoted by $\Delta_x u$. Similarly, $\Delta_y u$ and $\Delta_z u$ express the increments of u due to changes respectively in y alone and in z alone. The total change in u due to simultaneous changes in all the independent variables is sometimes denoted by Δu; so that if $u = f(x, y, z)$,

$$\Delta u = \frac{\Delta_x u}{\Delta x} \cdot \Delta x + \frac{\Delta_y u}{\Delta y} \cdot \Delta y + \frac{\Delta_z u}{\Delta z} \cdot \Delta z + \epsilon,$$

where ϵ is an infinitesimal of an order higher than the first.

The partial derivatives of u with respect to x, y, and z are denoted by $D_x u$, $D_y u$, and $D_z u$, and the sign $\doteq$ placed between a variable and a constant is used to show that the former is to be made to approach the latter as its limit. In those cases where it is desirable to draw attention to the fact that a certain derivative is total, the differential notation $\frac{du}{dx}$ is used.

view will find what they have learned here useful when they take up standard works on the subject.

My sincere thanks are due to H. N. Wheeler, A.M., who has read much of the manuscript of the following pages and all of the proof, and to Dr. E. H. Hall, who has examined parts of Chapters iv. and v. and helped me with various suggestions. I am indebted to other friends also, and among them to Mr. W. A. Stone for the use of some of his problems.

The reader who wishes to get a thorough knowledge of the properties of the Newtonian Potential Function and of its applications to problems in Electricity is referred to the following works, which, with others, I have consulted and used in writing these notes.

Betti: Teorica delle Forze Newtoniane e sue Applicazioni all' Elettrostatica e al Magnetismo.

Clausius: Die Potentialfunction und das Potential.

Cumming: An Introduction to the Theory of Electricity.

Chrystal: The article "Electricity" in the Ninth Edition of the Encyclopædia Britannica.

Dirichlet: Vorlesungen über die im umgekehrten Verhältniss des Quadrats der Entfernung wirkenden Kräfte.

Gauss: Allgemeine Lehrsätze in Beziehung auf die im verkehrten Verhältnisse des Quadrates der Entfernung wirkenden Anziehungs- und Abstossungskräfte. Also other papers to be found in Volume V. of his Gesammelte Werke.

Green: An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism.*

Mascart: Traité d'Electricité Statique. Also Wallentin's translation of the same work into German, with additions.

---

*A copy of the original edition of this paper is to be found in the Library of Harvard University, Gore Hall, Cambridge. The paper has been reprinted by Ferrers in "The Mathematical Papers of George Green," and by Thomson in Crelle's Journal.

Mascart et Joubert: Leçons sur l'Electricité et le Magnetisme. Also Atkinson's translation of the same work into English, with additions.

Mathieu: Théorie du Potential et ses Applications à l'Electrostatique et au Magnétisme.

Maxwell: An Elementary Treatise on Electricity. A Treatise on Electricity and Magnetism.

Minchin: A Treatise on Statics.

C. Neumann: Untersuchungen über das Logarithmische und Newton'sche Potential.

Riemann: Schwere, Electricität und Magnetismus, edited by Hattendorff.

Schell: Theorie der Bewegung und der Kräfte.

Thomson: Reprint of Papers on Electrostatics and Magnetism.

Thomson and Tait: A Treatise on Natural Philosophy.

Todhunter: A Treatise on Analytical Statics.

Watson and Burbury: The Mathematical Theory of Electricity and Magnetism.

Wiedemann: Die Lehre von der Electricität.

TABLE OF CONTENTS.

CHAPTER I.

THE ATTRACTION OF GRAVITATION.