

**PRIMARY ELEMENTS OF  
ALGEBRA:  
FOR COMMON SCHOOLS  
AND ACADEMIES**

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Primary elements of algebra: for common schools and academies by Joseph Ray

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**JOSEPH RAY**

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ALGEBRA:  
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AND ACADEMIES**



ECCLECTIC EDUCATIONAL SERIES.

*NEW ELEMENTARY ALGEBRA.*

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PRIMARY ELEMENTS

OF

**A L G E B R A,**

FOR

COMMON SCHOOLS AND ACADEMIES.

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By JOSEPH RAY, M. D.,

LATE PROFESSOR OF MATHEMATICS IN WOODWARD COLLEGE.

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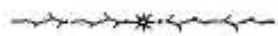
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## PREFACE.

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THE object of the study of Mathematics is two fold—the acquisition of useful knowledge, and the cultivation and discipline of the mental powers. A parent often inquires, "Why should my son study Mathematics? I do not expect him to be a surveyor, an engineer, or an astronomer." Yet, the parent is very desirous that his son should be able to reason correctly, and to exercise, in all his relations in life, the energies of a cultivated and disciplined mind. This is, indeed, of more value than the mere attainment of any branch of knowledge.

The science of Algebra, properly taught, stands among the first of those studies essential to both the great objects of education. In a course of instruction properly arranged, it naturally follows Arithmetic, and should be taught immediately after it.

In the following work, the object has been to furnish an elementary treatise, commencing with the first principles, and leading the pupil, by gradual and easy steps, to a knowledge of the elements of the science. The design has been, to present these in a brief, clear, and scientific manner, so that the pupil should not be taught merely to perform a certain routine of exercises mechanically, but to understand the *why* and the *wherefore* of every step. For this purpose, every rule is demonstrated, and every principle analyzed, in order that the mind of the pupil may be disciplined and strengthened so as to prepare him, either for pursuing the study of Mathematics intelligently, or more successfully attending to any pursuit in life.

Some teachers may object, that this work is too simple, and too easily understood. A leading object has been, to make the pupil feel, that he is not operating on unmeaning symbols, by means of arbitrary rules; that Algebra is both a rational and a practical subject, and that he can rely upon his reasoning, and the results

of his operations, with the same confidence as in arithmetic. For this purpose, he is furnished, at almost every step, with the means of testing the accuracy of the principles on which the rules are founded, and of the results which they produce.

Throughout the work, the aim has been to combine the clear explanatory methods of the French mathematicians with the practical exercises of the English and German, so that the pupil should acquire both a practical and theoretical knowledge of the subject.

While every page is the result of the author's own reflection, and the experience of many years in the school-room, it is also proper to state, that a large number of the best treatises on the same subject, both English and French, have been carefully consulted, so that the present work might embrace the modern and most approved methods of treating the various subjects presented.

With these remarks, the work is submitted to the judgment of fellow laborers in the field of education.

WOODWARD COLLEGE, *August*, 1848.

In this NEW ELECTROTYPE EDITION, the whole volume has been subjected to a careful and thorough revision. The oral problems, at the beginning, have been omitted; the number of examples reduced, where they were thought to be needlessly multiplied; the rules and demonstrations abridged; other methods of proof, in a few instances, substituted; and questions for GENERAL REVIEW introduced at intervals, and at the conclusion. It is confidently believed that these modifications, while they do not impair the integrity or change the essential features of the book, will materially enhance its value, and secure the approbation of all intelligent teachers.

*March*, 1866.

TO TEACHERS.—The following subjects may be omitted by the younger pupils, and passed over by those more advanced, until the book is reviewed: Observations on Addition and Subtraction, Articles 60—64; the greater part of Chapter II.; supplement to Simple Equations, Articles 164—177; properties of the Roots of an Equation of the Second Degree, Articles 215—217.

The pupil should be exercised in the solution of examples, until the principles are thoroughly understood; and, in the review, he should be required to demonstrate the rules on the blackboard.



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# ELEMENTS OF ALGEBRA.

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## I. DEFINITIONS.

NOTE TO TEACHERS.—Articles 1 to 15 may be omitted until the pupil reviews the book.

**Article 1.** In **Algebra**, quantities are represented by letters of the alphabet.

**2.** **Quantity** is any thing that is capable of increase or decrease; as, numbers, lines, space, time, etc.

**3.** Quantity is called *magnitude*, when considered in an undivided form; as, a quantity of water.

**4.** Quantity is called *multitude*, when made up of individual and distinct parts; as, three cents, a quantity composed of three single cents.

**5.** One of the single parts of which a quantity of multitude is composed, is called the *unit of measure*; thus, 1 cent is the *unit of measure* of the quantity 3 cents.

The *value* or *measure* of any quantity is the number of times it contains its unit of measure.

**6.** In quantities of magnitude, where there is no natural unit, it is necessary to fix upon an artificial unit as a standard of measure; then, to find the value of the quantity, we ascertain how many times it contains its *unit of measure*. Thus,

To measure the length of a line, take a certain assumed

REVIEW.—1. How are quantities represented in Algebra? 2. What is quantity? 3. When called magnitude?

4. When multitude? 5. What is the unit of measure? 6. How find the value of a quantity when there is no natural unit?