A CHAPTER IN THE INTEGRAL CALCULUS

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649237470

A Chapter in the Integral Calculus by A. G. Greenhill

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

A. G. GREENHILL

A CHAPTER IN THE INTEGRAL CALCULUS

Trieste

A CHAPTER

IN THE

INTEGRAL CALCULUS.

BY

A. G. GREENHILL, M.A.,

PROFESSOR OF MATHEMATICS TO THE SENIOR CLASS OF ARTILLERY OFFICERS, WOOLWICH
EXAMINER IN MATHEMATICS IN THE UNIVERSITY OF LONDON.

LONDON:
FRANCIS HODGSON, 89 FARRINGDON STREET, E.C.

1888.

LONDON:
PRINTED BY C. F. HODGSON AND SON,
GOUGH SQUARE, FLEET STREET.

PREFACE.

The present pamphlet is intended to be used by mathematical students as supplementary to an ordinary treatise on the Integral Calculus.

A method of Integration is proposed which makes this operation, always of a tentative nature, more systematic and certain; and in this method the hyperbolic functions are used freely in conjunction with the circular trigonometrical functions.

Considering their importance in Applied Mathematics, the hyperbolic functions have not received adequate treatment in ordinary text-books; to illustrate this importance, a digression has been made on their principal properties, illustrated by examples of their application.

The recent Cambridge examination papers have been consulted for examples, to exhibit the methods of integration explained in this pamphlet, which, it is hoped, will prove useful and interesting to the mathematical student.

CONTENTS.

A CHAPTER IN THE INTEGRAL CALCULUS.

Introduction.

PRESUMING that the reader is already familiar with the general methods of Integration, as given in text-books on the Integral Calculus, it is proposed in the following pages to explain how the process of Integration may in some cases be made more systematic, so that the result of the integration may be more readily written down, when the function to be integrated is once presented for that purpose.

The process of Integration is necessarily of a tentative nature, depending on a previous knowledge of Differentiation; and in general the most convenient order of the mental operations required for the integration of a given function will be found to be:

(i.) To guess the function required for the integration;
(ii.) To assign the argument of this function;
(iii.) To write down the proper constant or numerical factors of the integrated function.

Of these three operations, the first is of the most fundamental importance, as depending on the principles of the Calculus, but it is the second operation which presents the greatest practical difficulty, while the third only requires verification by a mental differentiation.

To lessen the difficulty of the second operation it is the object of these pages to show that it is convenient to take, either the function to be integrated, or *constituents* of this function, as the argument of the integrated function; for instance, when $\int \sec x \, dx$ is required, to express the result as a function of $\sec x$, and when

$$\int \frac{dx}{(x-p)\sqrt{(ax^2+2bx+c)}}, \quad \text{or} \quad \int \frac{Mx+N}{(Ax^2+2Bx+C)\sqrt{(ax^2+2bx+c)}}\, dx,$$

is required, to express the result as a function of

$$y = \frac{\sqrt{(ax^2+2bx+c)}}{x-p}, \quad \text{or} \quad \sqrt{\left(\frac{ax^2+2bx+c}{Ax^2+2Bx+C}\right)},$$

by changing the independent variable from x to y by these substitutions in the ordinary manner; and generally, to express $\int \phi(x)\, dx$ as a function of $\phi(x)$, or of the constituents of $\phi(x)$.