

# **ELEMENTS OF PLANE GEOMETRY**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649096466

Elements of plane geometry by Franklin Ibach

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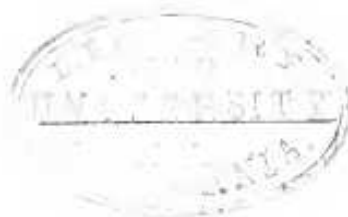
**ELEMENTS OF  
PLANE GEOMETRY**



**Butler's Series of Mathematics.**

**ELEMENTS**  
**OF**  
**PLANE GEOMETRY.**

**BY**  
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**PHILADELPHIA:**  
**E. H. BUTLER & CO.**

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Entered, according to Act of Congress, in the year 1882, by  
E. H. BUTLER & CO.  
In the Office of the Librarian of Congress, at Washington, D. C.

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## PREFACE.

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THIS little volume has been prepared with a view to furnish a suitable text-book on Plane Geometry for Grammar Schools, Preparatory Schools, etc.

A simple method of designating angles has been adopted, and recognized symbols have been freely used in the demonstrations, thus bringing the several steps closely together and enabling the student to master the argument with ease. The reasons on which the steps of an argument depend are not formally given, but are referred to by numbers indicating the sections in which they are found: it is believed that the pupil will impress the principles most firmly on his mind by frequency of reference.

No valid objection can be offered against the algebraic form of which some of the demonstrations partake, for most of the axioms laid down are nothing more than properties of the equation.

No apology is deemed necessary for the application of the *Infinitesimal* method: it has been employed whenever it gave directness, brevity, and simplicity to the demonstration.

At the close of each book, except the second, a collection of theorems and problems has been placed for the purpose of giving the pupil an opportunity to exercise his originality in demonstration and construction. A proper use of these exercises will do much toward stimulating thought and awakening a spirit of invention in the pupil.

During the preparation of this treatise, DIESTERWEG'S "Elementare Geometrie" and most of our American treatises have been freely consulted.

And now, this little work is respectfully submitted to the educational public, in the hope that it may at least merit a careful perusal.

F. IBACH.

PHILADELPHIA, PA., May, 1882.

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#### NOTE TO THE TEACHER.

In recitation, when studying a book for the first time, the pupil should be required to draw the diagram accurately and write the demonstration neatly on the blackboard.

Being called upon to recite, he enunciates the proposition and gives the demonstration, pointing to the parts of the diagram as reference is made to them.

In review, the diagram only should be put on the board.

# INTRODUCTION.

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## SUBJECT-MATTER.

THE accompanying diagram represents a block of granite,—a *physical solid*, of regular form.

Such a block has six flat faces, called *Surfaces*. It has also twelve sharp edges in which these surfaces meet, called *Lines*.

It has, besides, eight sharp corners in which these lines meet, called *Points*.

If the block be removed, we can imagine the space which it filled to have the same shape and size as the block. This limited portion of space, which has length, breadth, and thickness, is called a *Geometrical Solid*. Its boundaries or *surfaces* separate it from surrounding space, and have length and breadth but no thickness. The boundaries of these surfaces are *lines*, and have length only. The limits of these lines are *points*, and have position only. We thus come in three steps from solids to points, which have no magnitude. Having thus acquired notions of solids, surfaces, lines, and points, we can easily conceive of them distinct from one another. It is of such *ideal* solids, surfaces, lines, and points that Geometry treats; and these in various forms, except points, are called *Geometrical Magnitudes* or *Magnitudes of Space*.

