ELIMINATION BETWEEN TWO UNKNOWN EQUATIONS WITH TWO UNKNOWN QUANTITIES, BY MEANS OF THE GREATEST COMMON DIVISOR

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ELIMINATION

BETWEEN

TWO UNKNOWN EQUATIONS WITH TWO UNKNOWN QUANTITIES,

BY MEANS OF

THE GREATEST COMMON DIVISOR.

ALSO,

ANALYSIS OF CURVES,

WITH AN APPLICATION TO

AN EQUATION OF THE FOURTH DEGREE.

BY

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PREFACE.

The following pages consist of two demonstrations, which are intended to explain and illustrate parts of the Course of Mathematics which are only imperfectly discussed in Elementary Text-Books:

1. *Elimination between two equations with two unknown quantities, by means of the greatest common divisor.*

2. *Analysis of Curves—with an application to the discussion of an equation of the fourth degree.*

The first article is taken principally from Garnier and De Fourcy; and the second is a translation of the admirable discussion of Lacroix, in his *Calcul Différentiel et Intégral.*

Virginia Military Institute,

May, 1842.

RESOLUTION, &c.

Resolution of two equations with two unknown quantities.—
Elimination by the Greatest Common Divisor.

1. The most general equation of the mth degree, between two unknown quantities x and y, contains all the terms in which the sum of the exponents of x and y does not exceed m. Its form may then be represented by the equation

$$x^m + Px^{m-1} + Qx^{m-2} + Rx^{m-3} \dots + Tx + u = o,$$

in which P, Q, R, &c. are functions of y, as follows:

P represents a polynomial of the first degree in y of the form $a + by$

Q represents a polynomial of the second degree in y of the form $c + dy + ey^2$

R represents a polynomial of the third degree in y of the form $f + gy + hy^2 + ly^3$.

&c. &c. &c.

the last co-efficient u, containing all the powers of y, from zero to m.

2. An equation thus formed is said to be a *complete* equation of the mth degree between two unknown quantities, and when any of its terms are wanting, it is called an *incomplete* equation.

3. Could we solve equations of every degree, the ordinary methods of elimination might be readily applied, to the solution of any system of m equations, with m unknown quantities; and we should, in general, obtain a determinate number of solutions. It would be only necessary to find the value of one of the unknown quantities in terms of the others, in one of the equations, and substitute this value in each of the other equations; there would result a new system of equations, with one less equation than were given, and with one unknown quantity less. By continuing this operation, we should obtain a single equation with but one unknown quantity. This equation is called the *final* equation, and serves to determine the values of the unknown quantity which it contains, which by substitution will make known the corresponding values of the others.

4. If the number of equations exceeded the number of unknown quantities, we could by the above method eliminate all the unknown quantities, and there would result one or more equations, containing only known terms, which would be *equations of condition* necessary to be fulfilled, in order that the given equations should not be incompatible with each other.

5. Should the number of unknown quantities exceed the number of equations, the question would be *indeterminate;* for by giving arbitrary values to as many of the unknown quantities as were in excess, we might determine the values

of the others by means of the given equations, and thus have as many different solutions as there were arbitrary values assumed.

6. But the difficulty of solving equations in general, has led algebraists to seek other methods of elimination than the one just mentioned, so as to obtain at once a single equation involving but one unknown quantity. Various methods have been used to determine this final equation, and that method is regarded as best, which leads to a final equation, whose roots make known all the values of the unknown quantity which it contains, which are compatible with the given equations, and no other values. The method by the *Greatest Common Divisor* is not free from the objection of leading to foreign roots, but it is found to be the most convenient in practice. We propose to explain this method.

7. Let

$$A = o \qquad B = o$$

be two equations involving x and y, and let β be any assumed value of y. If we substitute this value in the place of y, in the given equations, there will result two equations,

$$A' = o \qquad B' = o$$

which contain only x and known quantities.

Now it is evident that β can only satisfy the given equations, when there exists at least one value of x, which will reduce the two quantities A' and B' to zero at the same time, that is, satisfy the equations