TABLES FOR FINDING THE INTERMEDIATE RATES OF INTEREST BETWEEN 3/4 AND 10 PER CENT., IN AN ANNUITY-CERTAIN

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TABLES

FOR FINDING THE

Intermediate Rates of Interest

BETWEEN $\frac{3}{4}$ AND 10 PER CENT.,

IN AN ANNUITY-CERTAIN,

The Present Value of **£1** *per annum (or period) for any number of years (or periods) not exceeding* **100** *being given.*

BY

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INTRODUCTION.

It is but just to state that the idea of constructing this Table was suggested by a careful study of Mr. D. J. McG. McKenzie's able and interesting article on a method of approximating the rate of interest in an Annuity-Certain, which appeared in the *Journal of the Institute of Actuaries*, No. CXXVIII., October, 1882.

The method is described as consisting "in finding the value of an "annuity at two rates of interest very close to each other; and when "this has been done with proper precautions, the required rate of "interest can be found with very great exactness by a simple "interpolation by first differences."

Mr. McKenzie, by numerous examples, showed in elaborate detail the working of his process, the exact rates of interest, true to six and eight places of decimals, being brought out with admirable precision, thus leaving nothing to be desired so far as extreme accuracy of results is concerned.

After an attentive and prolonged scrutiny of these examples, and a considerable number of experiments, it became evident that by means of the accompanying Table it was possible, in a very simple manner, readily to find the rate of interest in an Annuity-Certain, with a degree of exactness amply sufficient for almost all, if not for all, practical purposes.

METHOD OF USING THE TABLE.

The present values of £1 per annum in this work have been taken from the Author's *Tables of Compound Interest*,* with the number of decimals reduced near the commencement of the Table from five to four, and subsequently from five to three places.

* Published by C. & E. Layton, London.

The peculiar feature of the Table is an extra column, containing a series of multipliers, by the aid of which the rate of interest in an Annuity-Certain (generally true within about one farthing per cent.) can be easily and expeditiously found from the present value of an Annuity of £1 for any number of years (or periods) not exceeding 100.

The mode of using these multipliers will be at once seen from the following examples :—

EXAMPLE 1.—An annuity of £200, payable half-yearly for 22 years, in Red Sea Annuities was purchased for £3,002. 12*s.*: required, the rate of interest.

$\frac{3002{\cdot}6}{100}$ = 30·026 Present value of £1 per annum.

		Rate of Interest.
Next less value, Col. 44 =	29·782	1·875
	·244	·042
Multiplier, Col. 44 =	·172	1·833 Answer.
	488	Half-Yearly.
	4148	
	·041968	

EXAMPLE 2.—Required, the half-yearly sinking fund that will replace £3,002. 12*s.*, the purchase-money in the above case, at the end of 22 years at 1·833 per cent. half-yearly rate of interest.

£100 as interest on £3,002. 12*s.* = 3·330 per cent.
See Example 1 1·833
1·497 per cent.

1·497 per cent. on £3,002. 12*s.* = £44·949, or £44. 19*s.* Answer.

It will be found that a half-yearly sinking fund of £44·949 would in 22 years, at 1·833 half-yearly rate of interest, amount to £3,001.

EXAMPLE 3.—The lease of a house for 50 years was sold for £6,500, the clear annual rent being £400: required, the rate of interest.

$\frac{6,500}{400}$ = 16·250 Present value of £1 per annum.

		Rate of Interest.
Next less value, Col. 50 =	16·041	5·875
	·209	·091
Multiplier, Col. 50 =	·434	5·784 Answer.
	836	
	627	
	836	
	·090706	

EXAMPLE 4.—Required, the annual sinking fund that will in 50 years, at 5·784 per cent., replace £6,500, the purchase-money in the preceding case.

£400 as interest on £6,500 = 6·154 per cent.
See Example 3 = 5·784
0·370 per cent.

0·370 per cent. on £6,500 = £24·05, or £24. 1*s.* Answer.

It will be found that an annuity of £24·05 or £24. 1*s.*, laid up at 5·784 per cent., would at the end of 50 years amount to £6,501.

EXAMPLE 5.—An annuity of £72 for 10 years cost £489. 12*s.* required, the rate of interest.

$\frac{489{\cdot}6}{72} = 6{\cdot}80000$
Next less value, Col. 10 = 6·78641
·01359
Multiplier, Col. 10 = 3·233
4077
4077
2718
4077
·04393647

Rate of Interest.
7·750
·044
7·706 Answer.

EXAMPLE 6.—£4,200 was the sum paid for a 37 years' lease of a house yielding a clear rent of £246: required, the rate of interest.

$\frac{4{,}200}{246} = 17{\cdot}073$
Next less value, Col. 37 = 16·988
·085
Multiplier, Col. 37 = ·440
3400
340
·037400

Rate of Interest.
4·875
·037
4·838 Answer.

EXAMPLE 7.—The present value of £1 per annum for 72 years is 21·788. What is the rate of interest?

21·788
Next less value, Col. 72 = 21·288
·500

Rate of Interest.
4·500
Deduct ½ Multiplier, Col. 72 = ·120
4·380 Answer.

EXAMPLE 8.—The present value of £1 per annum for 91 years is 32·070. Find the rate of interest.

	32·070		Rate of Interest.
Next less value, Col. 91 =	31·070		3·000
	1·000	Deduct Multiplier, Col. 91 =	·116
			2·884 Answer.

In none of the foregoing instances, which have been taken hap-hazard, do the rates of interest given in the answers differ to the extent of a farthing from the exact rates (as found by seven figure logarithms), and as equally satisfactory results have been obtained in an extensive series of trials, it is manifest that the Table may with safety be used in all analogous cases.

In the comparatively rare financial transactions involving millions of pounds sterling, it may, no doubt, be desirable to find the rate of interest true to four or five places of decimals. The following examples taken (for convenience in verifying the final results) from those carefully worked out by Mr. McKenzie, will show that the Table, supplemented by a slight logarithmic operation, is capable of giving the rate of interest with the requisite amount of accuracy.

EXAMPLE 9.—The present value of an annuity of £40 for 60 years being £500: required, the rate of interest (Baily's *Doctrine of Interest*, page 131).

$\frac{500}{40}$ =	12·500		Rate of Interest.
Next less value, Col. 60 =	12·377		8·000
	·123		·082
Multiplier, Col. 60 =	·667		7·918
	861		− ·000727
	738		7·917273 Answer.
	738		7·917277 True.
	·082041		·000004 Error.

$(1+i) = 1{\cdot}07918$ and Log $1{\cdot}07918 = 0{\cdot}0330939$

		60	
		1·9856340	
Colog ·07918 =	1·1013845	=	12·62945
	$\bar{1}$·1157505	=	·13054
			12·49891
			12·50000
			− ·00109
		Multiplier, Col. 60 =	·667
			·00072703

EXAMPLE 10.—The present value of £1 for 50 years being 22·61794 : required, the rate of interest.

		Rate of Interest.	
	22·618	3·750	
Next less value, Col. 50 =	22·434	·046	
	·184	3·704	
Multiplier, Col. 50 =	·249	− ·000227	
	1656	3·703773	Answer.
	736	3·703775	True.
	368	·000002	Error.
	·045816		

$(1+i) = 1{\cdot}03704$ and Log $1{\cdot}03704 = 0{\cdot}0157955$

	50		
	0·7897750		
Colog ·03704 =	1·4313290	=	26·99784
	0·6415540	=	4·38081
			22·61703
			22·61794
			− ·00091
			·249
			·00022659

EXAMPLE 11.—A loan is issued at the price of 78 per cent., redeemable at par in 30 years by an accumulative sinking fund. The annuity is 6·505144 and $a_{\overline{30}|}$ is 11·99051 the true value (*Journal of the Institute of Actuaries*, xix., 89–94) : required, the rate of interest.

		Rate of Interest.	
	11·991	7·375	
Next less value, Col. 30 =	11·956	·030	
	·035	7·345	
Multiplier, Col. 30 =	·844	+ ·000262	
	4220	7·345262	Answer.
	2532	7·345267	True.
	·029540	0·000005	Error.

$(1+i) = 1{\cdot}07345$ and Log $1{\cdot}07345 = 0{\cdot}0307818$

	30		
	0·9234540		
Colog ·07345 =	1·1340082	=	13·61470
	0·2105542	=	1·62388
			11·99082*
			11·99051
			+ ·00031
			·844
			+ ·00026164

* N.B.—The present value of £1 per annum for 30 years at 7·345 per cent. being ·00031 in excess of 11·99051, the final product, ·00026164, becomes additive instead of subtractive as in cases 9 and 10.