

**SOLID GEOMETRY  
DEVELOPED BY  
THE SYLLABUS  
METHOD, PP. 193-403**

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Solid Geometry Developed by the Syllabus Method, pp. 193-403 by Eugene Randolph Smith & William H. Metzler

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**EUGENE RANDOLPH SMITH & WILLIAM H. METZLER**

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# SOLID GEOMETRY

DEVELOPED BY THE

## SYLLABUS METHOD

BY

EUGENE RANDOLPH SMITH, A.M.

HEADMASTER, THE PARK SCHOOL, BALTIMORE, MARYLAND  
(FORMERLY HEAD OF THE DEPARTMENT OF MATHEMATICS, POLYTECHNIC  
PREPARATORY SCHOOL, BROOKLYN, N.Y.)

IN CONSULTATION WITH

WILLIAM H. METZLER, Ph.D.

DEAN OF THE GRADUATE SCHOOL, AND HEAD OF THE DEPARTMENT  
OF MATHEMATICS, SYRACUSE UNIVERSITY

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## PREFACE

THE author has always believed that the teaching of mathematics should encourage original thinking on the part of the pupils, and that a maximum of thought is difficult to obtain when the pupil is furnished with a text containing the proofs of the geometrical theorems in synthetic form. He has been using the heuristic method for thirteen years, and has proved to his own satisfaction that the average pupil can enter intelligently into the class development of new propositions, and, after a certain amount of training, can analyze propositions of ordinary difficulty with little or no help from the teacher.

This book has been written to encourage what is often called the "genetic," "heuristic," or "syllabus" method of teaching, but what is, in effect, simply the development of new work, partly in class, the rest by assignment as original exercises for preparation before the recitation. It is intended to contain all that a pupil really needs, except that which the teacher, and only the teacher, can best supply. While the proofs are not in the book in full and formal fashion, the author has given as much analysis, suggestion, and guidance as he feels to be wise. In fact, he believes that even this amount of help should often not be used until after the class discussion has covered the topic.

The list of propositions is based on the recommendations of the various committees, and especially on that of the National Geometry Committee of Fifteen. An attempt

has been made to reduce the list to a *pedagogical minimum*, without injuring the subject by over-reduction. The space thereby saved has been devoted to valuable matter not often found in geometries, such as the preliminary chapter, the detailed summaries, the Appendix, and the college examination questions.

Solid geometry is an especially promising field for heuristic methods because the pupils studying it are of sufficient maturity to enable them to think with some assurance and originality. If a pupil is ever to obtain the proper training in logical methods of thinking, it ought not to be longer delayed. Besides this, the results in solid geometry in schools and colleges show that former methods have not proved entirely satisfactory. This condition is probably due to the fact that the student is often plunged into the subject with little or no understanding of the figures with which he is to deal, the method he should use in studying them, or the relation of this new subject to the mathematics that has gone before. This book attempts to better this condition by introducing a preliminary chapter that *shows the relation between plane geometry and solid geometry*, and by building up the solid geometry figures in easy gradation from points, lines, and planes. This arrangement will give the pupil a clear understanding of the formation of the figures with which he is dealing and will lead him, in progressive stages, from the less complicated figures and proofs to those of greater difficulty. Incidentally, many of the difficult parts of the subject have been simplified, and the pupil is given a broad working knowledge of the properties and formulas of solids.

There are three combinations of related parts of the subject into single sections: prisms and cylinders, pyra-



mids and cones, and polyhedral angles and spherical polygons. The simplification of the subject by these combinations is quite marked, and will be evident on inspection of the sections in question. In the last case, it gives opportunity for some consideration of the way in which each can be made to depend on the other.

Some use has been made of two of the most powerful propositions of solid geometry, Cavalieri's Theorem, and the Prismatoid Formula, but only as alternate methods. The National Geometry Committee recommends that these two theorems be used but not proved. The Prismatoid Formula is used for spherical segments.

The use of limits and the incommensurable case has been left to the discretion of the teacher. The best present-day thought seems to favor the omission of as much of this work as possible.

The exercises are of wide variety in subject and difficulty, and include a set of about two hundred and seventy-five college examination questions chosen from recent papers set by various colleges, the New York State Board of Regents, and the College Entrance Examination Board. The exercises are arranged, some after the theorems to which they apply, and the rest in general sets.

While the author has made use of all the knowledge of solid geometry that his study and teaching have given him, still, as this text was written chiefly from classroom experiment, it is impossible to give credit specifically to authors from whom the germs of ideas may have been received. He is greatly indebted to Dr. William H. Metzler of Syracuse University, for his painstaking reading of the manuscript and for his valuable criticisms. Mr. Howard F. Hart of the Montclair High School, who has been trying in his classes some of the ideas that appear in

this book, has given the author the benefit of his experience with them, as well as a number of suggestions on other parts of the book. Mr. Clarence P. Scoboria of the Polytechnic Preparatory School, Brooklyn, also has aided in the preparation of the book by using the manuscript in class, and so making a practical test of the ideas represented. Besides those mentioned, the author wishes to express his obligation to many others who have taken an active interest in the preparation of the book.

EUGENE RANDOLPH SMITH.

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