

**A TRACT ON THE
ADDITION OF
ELLIPTIC AND HYPER-
ELLIPTIC INTEGRALS**

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A Tract on the Addition of Elliptic and Hyper-Elliptic Integrals by Michael Roberts

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MICHAEL ROBERTS

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BY

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P R E F A C E.

THIS little work was written with the view of presenting in a simple and elementary way the fundamental properties of those transcendents which, in the ascending order of classification, immediately succeed elliptic functions. I have adopted as the basis of my investigations the following memoirs of Jacobi, "Considerationes generales de transcendentibus Abelianis" (*Crelle*, vol. 9, p. 394), "De functionibus quadrupliciter periodicis, quibus theoria transcendentium Abelianarum innititur" (*Crelle*, vol. 13, p. 55), "Demonstratio nova theorematis Abeliani" (*Crelle*, vol. 24, p. 28), and of the greater part of these memoirs much of what appears in the following pages may be regarded as a free translation. By the employment of the method given in the last of the above-mentioned memoirs, I have con-

structed a trigonometry of the functions in question, by which the reader is introduced to the study of them in the same way as he has already become acquainted with the properties of circular functions by the formulæ of ordinary trigonometry. Since the appearance of Jacobi's papers, the subject has received large developments at the hands of Riemann, Weierstrass, and other distinguished mathematicians, and I venture to hope that what I have put together may invite the attention of mathematical students to the higher theories and more abstract principles of the German analysts.

The perfect uniformity of treatment of elliptic and hyper-elliptic functions, by which their fundamental properties are derived from the employment of Jacobi's method, is shewn in the second, third, fourth, and fifth chapters; the sixth chapter is devoted to an accurate examination of those cases where the first class of hyper-elliptic integrals depends on elliptic functions, a question which appears, ever since the subject was started, to have been invested with interest, and the analogues of Fagnano's theorem, which I have given in the seventh chapter, bear

out the well-known analogy between conic sections and the lines of curvature of an ellipsoid.

I have to acknowledge, in the first place, my obligations to Mr. Cathcart, Fellow of Trinity College, Dublin, for the readiness with which he undertook the revision of the proof sheets, by which greater freedom from typographical errors has been attained than could have been secured without such aid, and also for suggestions of which I gladly availed myself.

I have also to thank the Board of Trinity College for their liberality in contributing to the expense of publication.

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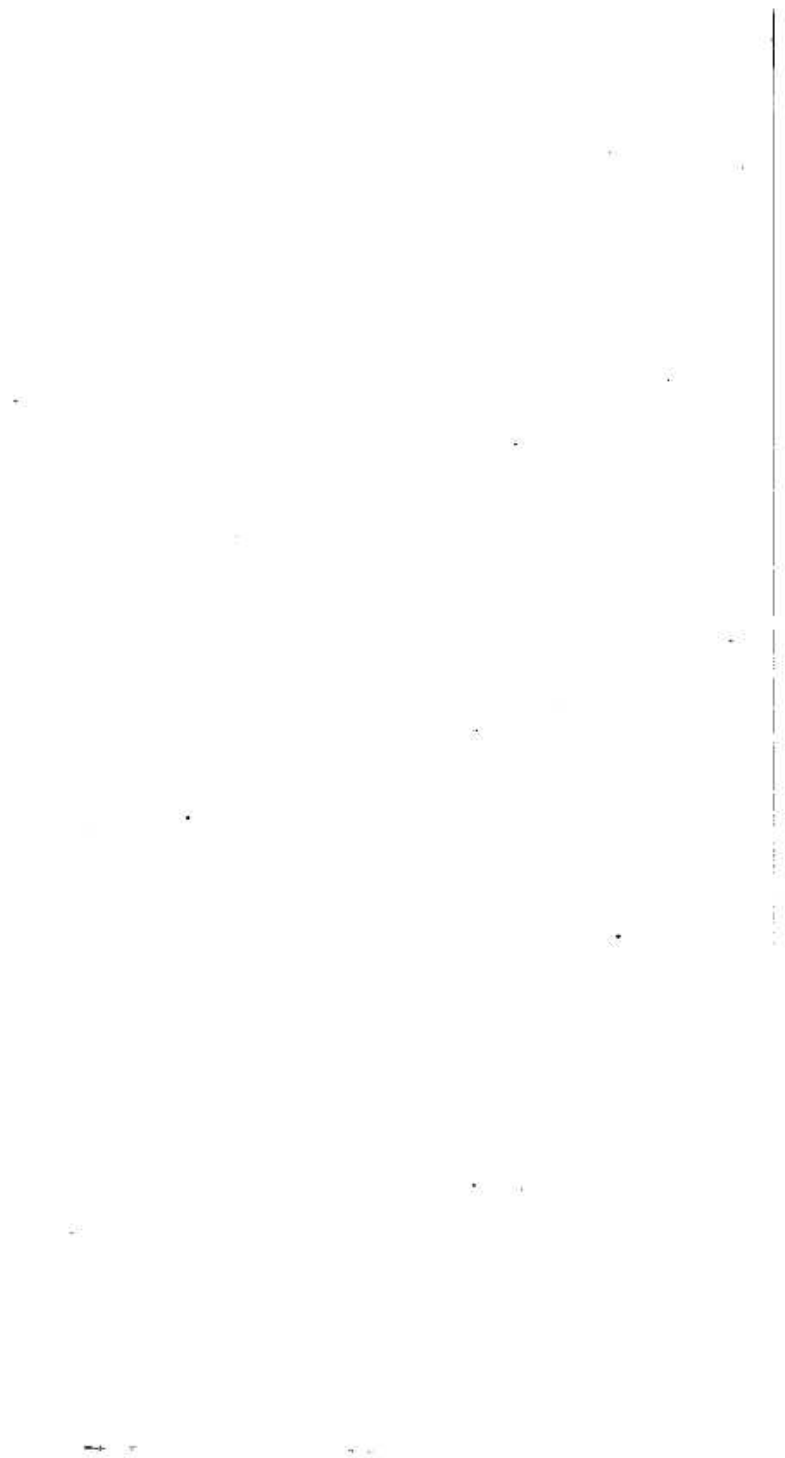


Figure 1. A 3D scatter plot showing the relationship between the number of species (S) and the number of individuals (N) for 10 different species. The plot is a 3D coordinate system with axes labeled S, N, and a vertical axis. The data points are labeled with species names: A, B, C, D, E, F, G, H, I, and J. Species A and B are clustered together at the top right. Species C and D are clustered together in the middle. Species E and F are clustered together at the bottom left. Species G, H, I, and J are scattered throughout the plot. A vertical line is drawn on the right side of the plot, and a horizontal line is drawn at the top. The plot is titled "Figure 1" and has a caption below it.

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