A TREATISE ON THE THEORY OF INVARIANTS

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OLIVER E. GLENN

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PREFACE

The object of this book is, first, to present in a volume of medium size the fundamental principles and processes and a few of the multitudinous applications of invariant theory, with emphasis upon both the nonsymbolical and the symbolical method. Secondly, opportunity has been taken to emphasize a logical development of this theory as a whole, and to amalgamate methods of English mathematicians of the latter part of the nineteenth century — Boole, Cayley, Sylvester, and their contemporaries — and methods of the continental school, associated with the names of Aronhold, Clebsch, Gordan, and Hermite.

The original memoirs on the subject, comprising an exceedingly large and classical division of pure mathematics, have been consulted extensively. I have deemed it expedient, however, to give only a few references in the text. The student in the subject is fortunate in having at his command two large and meritorious bibliographical reports which give historical references with much greater completeness than would be possible in footnotes in a book. These are the article "Invariantentheorie" in the "Enzyklopädie der mathematischen Wissenschaften" (I B 2), and W. Fr. Meyer's "Bericht über den gegenwärtigen Stand der Invariantentheorie" in the "Jahresbericht der deutschen Mathematiker-Vereinigung" for 1890–1891.

The first draft of the manuscript of the book was in the form of notes for a course of lectures on the theory of invariants, which I have given for several years in the Graduate School of the University of Pennsylvania.

The book contains several constructive simplifications of standard proofs and, in connection with invariants of finite groups of transformations and the algebraical theory of ternariants, formulations of fundamental algorithms which may, it is hoped, be of aid to investigators.

While writing I have had at hand and have frequently consulted the following texts:

CLEBSCH, Theorie der binären Formen (1872).
CLEBSCH, LINDEMANN, Vorlesungen über Geometrie (1875).
DICKSON, Algebraic Invariants (1914).
DICKSON, Madison Colloquium Lectures on Mathematics (1913). I. Invariants and the Theory of Numbers.
ELLIOTT, Algebra of Quantics (1895).
FAÀ DI BRUNO, Théorie des formes binaires (1876).
GORDAN, Vorlesungen über Invariantentheorie (1887).
GRACE and YOUNG, Algebra of Invariants (1903).
W. FR. MEYER, Allgemeine Formen und Invariantentheorie (1909).
W. FR. MEYER, Apolarität und rationale Curven (1883).
SALMON, Lessons Introductory to Modern Higher Algebra (1859; 4th ed., 1885).
STUDY, Methoden zur Theorie der ternären Formen (1889).

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PHILADELPHIA, PA.

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