

**THE PHILOSOPHY OF
MATHEMATICS: WITH SPECIAL
REFERENCE
TO THE ELEMENTS OF GEOMETRY
AND THE INFINITESIMAL METHOD**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649670451

The Philosophy of Mathematics: With Special Reference to the Elements of Geometry and the Infinitesimal Method by Albert Taylor Bledsoe

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W. A. Butler

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INFINITESIMAL METHOD.

BY

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PHILADELPHIA:
J. B. LIPPINCOTT COMPANY.
1886.

Entered according to Act of Congress, in the year 1887, by
J. B. LIPPINCOTT & CO.,
In the Clerk's Office of the District Court of the United States, for the Eastern
District of Pennsylvania.

LIPPINCOTT'S PRESS,
PHILADELPHIA.

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THE PHILOSOPHY OF MATHEMATICS.

CHAPTER I.

FIRST PRINCIPLES OF THE INFINITESIMAL METHOD— THE METHOD OF EXHAUSTION.

THE student of mathematics, on passing from the lower branches of the science to the infinitesimal analysis, finds himself in a strange and almost wholly foreign department of thought. He has not risen, by easy and gradual steps, from a lower into a higher, purer, and more beautiful region of scientific truth. On the contrary, he is painfully impressed with the conviction, that the continuity of the science has been broken, and its unity destroyed, by the influx of principles which are as unintelligible as they are novel. He finds himself surrounded by enigmas and obscurities, which only serve to perplex his understanding and darken his aspirations after knowledge. That clearness of evidence, which is the boast of the mathematics, and which has hitherto cheered and stimulated his exertions, forsakes him as soon as he enters on the study of the infinitesimal calculus, and the darkness of doubt settles on his path. If, indeed, he does not

abandon the study in disgust or despair, as thousands have done, he pursues it for the sake of a diploma or a degree, or from some less worthy motive than the love of science. He certainly derives from it comparatively little advantage in the cultivation of his intellectual powers; because the dark and unintelligible processes he is required to perform scarcely demand a natural exercise of them.

These disadvantages of the study are due, for the most part, to the manner in which the calculus is usually taught. In most elementary works on the differential calculus, the first principles of the science are not set forth at all, or else so imperfectly defined as to mislead the student from the clear path of mathematical science into a region of clouds and darkness. I have frequently made the experiment with some of the best of such works. I have more than once put them into the hands of a class of from ninety to a hundred students, among whom there were mathematical minds of no ordinary power, and required them to tell me what are the first principles of the infinitesimal method or calculus. Yet, after having read and mastered the first chapter, which, of course, contained a discussion of "First Principles," not one of them had acquired the least notion of what those principles are. Not one of them could even name the first principles of the science, much less define them. In this respect, the most capable and diligent members of the class were on a perfect level with the most stupid and indolent. Indeed, if the authors of the books themselves knew what the first principles of the calculus are, they were very careful not to unveil their knowledge.

Now, the very first condition of the existence of a mathematical science *as such* is, that its first principles shall be so clear and so perfectly defined that no one could mistake them. But even this primary and indispensable condition is not fulfilled by most of the treatises or text-books on the infinitesimal analysis. Hence this analysis, as usually developed in books for the instruction of beginners, is still in a semi-chaotic state. If, then, we would introduce anything like the order, harmony, and beauty of real mathematical science into the transcendental analysis, the first step to be taken is to exhibit its first principles in a clear and unmistakable light. My object in this work is to contribute all in my power toward so desirable a result; or, in other words, to render as clear as possible the fundamental principles of the higher calculus, from which the whole science should be seen to flow in the form of logical consequence, and that, too, as clearly as the light of day flows from the sun. Much has already been done in this direction; far—far more than has been appropriated by the so-called teachers of the science. Hence I shall have frequent occasion to avail myself of the labors of others; but I shall never do so without an explicit acknowledgment of my obligation to them.

In the prosecution of this design, I shall trace the rise and progress of the infinitesimal analysis from the first appearance of its elements in the Greek geometry to the present day. This will enable us to see, the more clearly, the exact nature of its methods, by showing us the difficulties it has had to encounter, and the precise manner in which it has surmounted them. It will also disclose, in a clear light, the merits of