

**LECTURES ON THE THEORY OF
PLANE CURVES; DELIVERED TO
POST-GRADUATE STUDENTS IN
THE UNIVERSITY OF CALCUTTA,
PART II, PP.140-350**

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SURENDRAMOCHAN GANGULI

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LECTURES
ON THE
THEORY OF PLANE CURVES

DELIVERED TO POST-GRADUATE STUDENTS
IN THE UNIVERSITY OF CALCUTTA

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PART II



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PREFACE

The subject of Analytical Geometry covers so extensive a field that it is by no means easy to decide what to omit and what to insert ; a word is therefore necessary to explain the plan adopted in this book, which, I trust, will prove a useful introduction to the higher branches of the subject and will facilitate the study of a variety of algebraic curves.

In the preparation of these lectures, I have endeavoured to present the subject in clear and concise terms to the student commencing a systematic study of the properties of algebraic curves, especially of Cubics and Quartics. In the portion of the book devoted to the discussion of cubic curves, I have not confined myself exclusively to the application of analytical methods, but have availed myself of the methods of Geometry whenever simplicity could be gained thereby. One prominent feature of the present work is that properties of cubic curves have been exhaustively discussed with special reference to points of inflexion and harmonic polars ; in this connection, canonical forms have at times been found of great use. A separate chapter has been allotted to the discussion of some special cubic curves of historic importance, and their most general properties ; but no systematic analysis has been attempted, lest the young student should feel embarrassed ; only

general characteristics of these curves have been outlined which will supply sufficient material for independent thinking in more advanced stages.

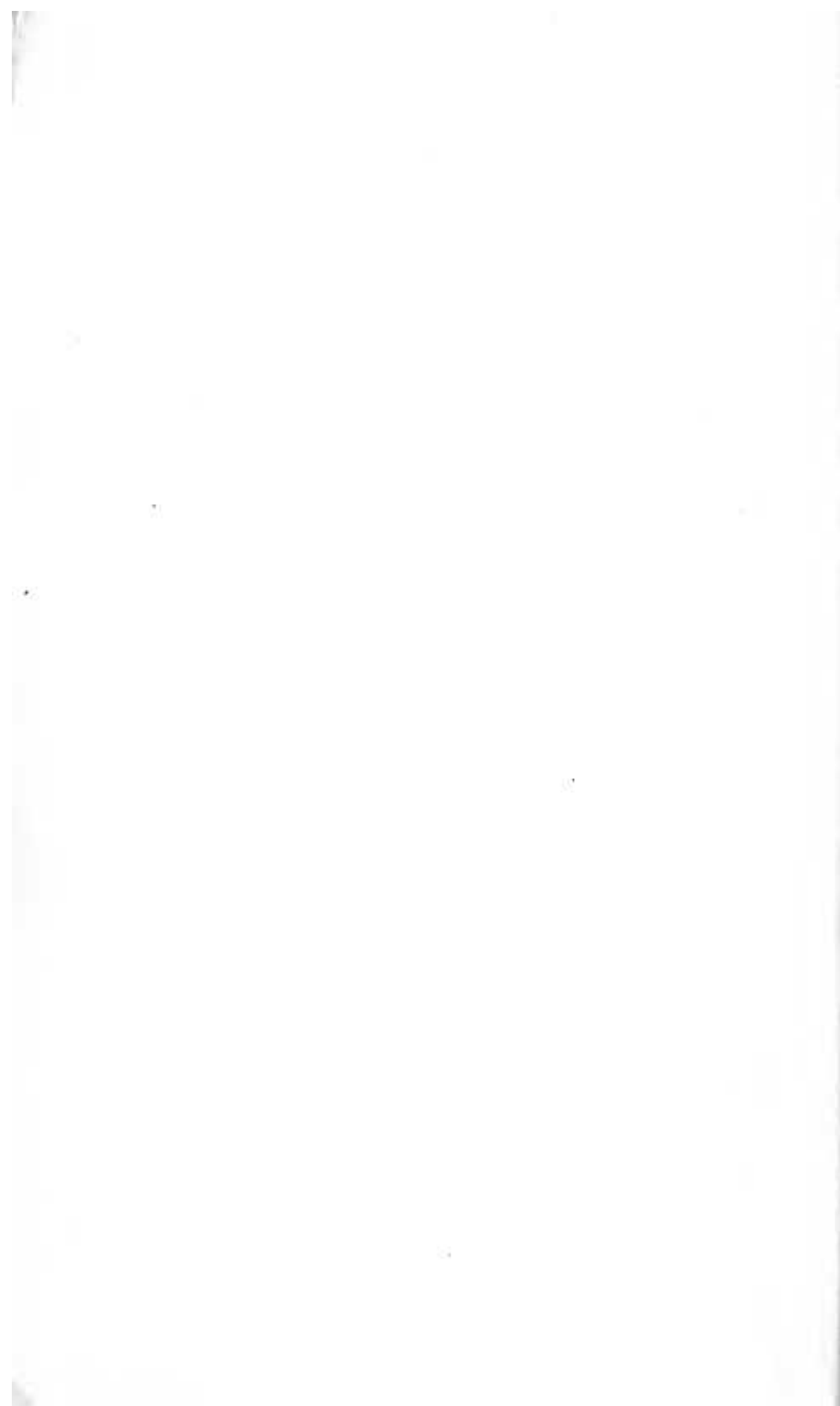
The subject of quartic curves is too extensive to be adequately considered in a small work like this. I have therefore confined the discussion chiefly to the most prominent characteristics of these curves. One chapter has been devoted to the consideration of bicircular quartics with special reference to their mode of generation. In fact, this chapter, together with a note in Appendix I, is mainly based on the well-known Memoir on bicircular quartics by Dr. Casey, published in the Transactions of the Royal Irish Academy 1869. Circular cubics have been studied with much advantage, regarded as degenerate bicircular quartics. In the last chapter are considered some well known quartic curves, most of which are bicircular or are cartesian. A similar consideration, as in the case of cubic curves, has led me to restrict my discourse only to the general properties of these curves. I have intentionally avoided the discussion of Roulettes, Cycloids, etc., reserving the topics for a future occasion. The reader who desires to study the subject from a higher standpoint can conveniently consult the following works—Clebsch—*Leçons sur la Géométrie*; Chasles—*Histoire de la Géométrie*. In Appendix II, a note on Trinodal Quartics has been inserted. This was communicated to me by Rai A. C. Bose, Bahadur, M.A., Controller of Examinations, University of Calcutta.

In studying singular points on cubics and quartics, I have retained the common phrase "non-singular" to designate a curve which has no double point or multiple point, although it has been pointed out by Prof. Basset that this is a misnomer; for, he says, Plücker had shown that all algebraic curves except conics possess singularities and accordingly he introduced the term "anautotomic" in preference to the phrase "non-singular" commonly in use.

In concluding this preface, I desire to say that, in addition to the works of authors cited in the preface to the first part, I have consulted, with much advantage, some notes on Cubics and Quartics furnished by my colleague Dr. H. D. Bagchi, M.A., Ph.D., and that I am indebted for some valuable hints to Rai A. C. Bose, Bahadur, M.A. Once more I must acknowledge myself in the highest degree indebted to Sir Asutosh Mookerjee, Kt., President of the Council of Post-Graduate Teaching in Arts, for his extreme kindness in encouraging me to revise these lecture-notes for the press, and to the authorities of the University of Calcutta for publishing them. Finally, I must thank the Staff of the Calcutta University Press, but for whose untiring energy and ready co-operation, the second part of the book could not have seen the light of day before December next.

University of Calcutta,
August, 1919.

S. M. GANGULI.



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