SOLID GEOMETRY, PP. 304-494

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PREFACE

During recent years the study of solid geometry has occupied a somewhat less commanding position in the mathematical curriculum than formerly. Important and essential as its subject matter is admitted to be, it has been little more than an appendage to plane geometry, both in the methods of its presentation and in its scientific results.

The authors of this text feel that the subject is much more vital than such a tendency would indicate. Not only are the bare truths gained from a study of solid geometry essential to the student of science, but through its medium a multitude of mathematical ideas can be presented and elucidated in a natural and convincing manner. In fact, no subject of elementary mathematics can be compared to solid geometry as a climax and capstone of mathematical study for the student who pursues the subject no farther. It not only utilizes and applies much that he has learned in other courses, but serves as a point of vantage from which may be gained many glimpses of scientific fields which he is not to enter.

In this text the authors have presented the subject in such form that a minimal course as prescribed by the colleges and the various examining boards may be covered. At the same time it affords at every turn a richness of suggestion and development for those who have the time and the inclination to do more than that minimum.

One of the important opportunities afforded by the study of solid geometry is that of using and developing the scientific

imagination. This text, through its hundreds of queries, aims to encourage the student to regard the subject not merely as a logical sequence of theorems but as a subject inviting reflection and the play of speculation. These queries should be used in class as a basis for discussion and will be found to render the more formal work not only more interesting but more intelligible. If time does not permit any attention to the queries, they may be omitted from the class assignments without disturbing the continuity of the subject. Their use, however, is strongly urged by the authors.

Exercises illustrating or dependent upon the various theorems are scattered throughout the text and afford as much drill of this kind as many teachers can profitably use. The collections at the end of each book may be regarded as supplementary. Great care has been exercised to provide a collection of originals that is fresh, interesting, not too difficult, but illustrating all parts of the subject.

The assumption of Cavalieri's Theorem as a basis for the theorems on measurement is the result of many years of classroom experience. The simplicity and power of this procedure should commend it both to teachers and to students.

The geometry of the sphere and its relation to plane geometry is also elaborated with care and in such a manner as to give the student an insight into the meaning of geometrical science.

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REFERENCES FROM PLANE GEOMETRY

POSTULATES AND AXIOMS

19. Postulate I. There is only one straight line through two points.

20. Postulate II. Any geometric figure may be moved from one place to another without changing its size or shape.

31. Axiom I. If equals are added to equals, the results are equal.

32. Axiom II. (1) Two numbers or magnitudes each equal to a third are equal to each other. (2) Two figures congruent to a third are congruent to each other.

36. Postulate III. All straight angles are equal.

39. Axiom III. If equals are divided by the same number, the results are equal.

41. Postulate IV. At a given point of a line, one and only one perpendicular can be drawn to the line.

45. Postulate V. The postulate of parallels. Through a given point outside a line, one line parallel to it exists, and only one.

51. Axiom IV. If equals are subtracted from equals, the results are equal.

65. Axiom V. A number may be substituted for its equal in any operation on numbers.

124. Axiom VI. If equals are multiplied by equals, the results are equal.

136. Axiom VII. The whole is greater than any of its parts.

137. Axiom VIII. If the first of three magnitudes is greater than the second and the second is greater than the third, the first is greater than the third.

139. Axiom IX. If the same number, positive or negative, is added to or subtracted from each member of an inequality, the results are unequal in the same order.

140. Axiom X. If both members of an inequality are multiplied or divided by the same positive number, the results are unequal in the same order.

141. Axiom XI. If the corresponding members of two or more inequalities which are in the same order are added, the sums are unequal in the same order.

142. Axiom XII. If unequals are subtracted from equals, the results are unequal in the reverse order.

146. Postulate VI. Any side of a triangle is less than the sum of the other two sides.

DEFINITIONS

15. Angle. A plane angle (symbol $\angle$) is the figure formed by two rays which meet.

16. Triangle. A triangle (symbol $\triangle$) is a portion of a plane bounded by three straight lines.

24. Congruence. Two geometric magnitudes are congruent if their boundaries can be made to coincide.

30. Isosceles triangle. An isosceles triangle is a triangle which has two equal sides.

37. Perpendicular. If one straight line cuts another so as to make any two adjacent angles equal, each line is perpendicular (symbol $\perp$) to the other.

43. Parallel lines. Parallel lines are lines that lie in the same plane and do not meet however far they are produced.

49. Hypotenuse. The hypotenuse of a right triangle is the side opposite the right angle.

52. Vertical angles. Two angles are vertical angles if the sides of one are the prolongations of the sides of the other.