TEXT-BOOK OF ELEMENTARY PLANE GEOMETRY

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649356447

Text-book of elementary plane geometry by Julius Petersen

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

www.triestepublishing.com

JULIUS PETERSEN

TEXT-BOOK OF ELEMENTARY PLANE GEOMETRY

Trieste

TEXT-BOOK

OF

ELEMENTARY PLANE GEOMETRY

BY

JULIUS PETERSEN,
PROFESSOR AT THE ROYAL POLYTECHNIC SCHOOL AT COPENHAGEN
AND MEMBER OF THE ROYAL DANISH SOCIETY OF SCIENCE.

TRANSLATED BY

R. STEENBERG Jr.

(AUTHORIZED ENGLISH EDITION)

LONDON
SAMPSON LOW, MARSTON, SEARLE & RIVINGTON
COPENHAGEN: ANDR. FRED. HÖST & SON
1880

Printed by Bianco Luno.

INTRODUCTION.

1. Every object which occurs in nature has numerous properties; to facilitate the investigation of which, one preliminarily endeavours to consider one at a time, independently of the others; the different investigations of the same kind are then collected, and thus the different sciences, *natural sciences*, are formed. Thus, if we examine a piece of chalk, we might ask about its origin and occurrence, its specific gravity and colour, the combination of its elements, its form, &c., and these questions will be answered respectively by Geology, Physics, Chemistry, and Geometry.

2. Geometry treats of the *form* without regard to the substance; when we speak of a sphere, we do not consider of what it is made, but only of the space which it occupies; every object occupies a space, which has extension in all directions; this is a geometrical *body*.

The boundary between a body and the surrounding space is called a *superficies* or the surface of the body; a superficies has no thickness.

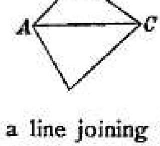
If a part of a superficies be cut off from the surrounding part, the boundary is called a *line*; a line has no breadth. If a part of a line be cut off, the limit is termed a *point*; it has no magnitude, but only denotes position.

3. When a point moves, it produces a line, similarly a superficies may be imagined as being produced through the movement of a line, a body through the movement of a superficies.

4. Let us imagine a body revolving round two fixed points, that is, moving so that two points in it do not change their places in space, we could then suppose a line to be drawn through these two points and through all the other points in the body, which during the movement do not change their places. Such a line is called a *straight line*. The straight line, therefore, is determined by two points, that is, that *through two given points there can only be drawn one straight line.* From this it follows again, that *two different straight lines can only have one point in common (a point of intersection)*, for if they had two points in common, they must coincide. A straight line can be imagined to be infinitely produced by the addition of other straight lines, each having two points in common with the preceding ones. Where it cannot be misunderstood line is often used for straight line.

A line of which no part is straight is called *curved*, a line composed of several straight lines is called a „*broken line*".

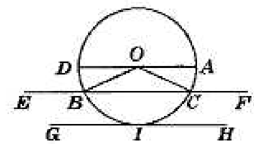
5. A plane superficies or *plane* is one in which every straight line lies wholly, when two of its points lie in it; the position of a plane is determined, when it contains three given points which are not in a straight line. A straight line and a plane are supposed infinitely produced, when the opposite is not stated; a straight line may, without being changed, be imagined to be made to slide along itself; a plane may, without being changed, be made to slide along itself or turned in itself round one of its points.



6. A *figure* is a limited part of a plane, its boundary is called *Perimeter*, when it is broken, *Periphery*, when it is curved. A *triangle* is contained by three straight lines (sides), a quadrilateral by four, a multilateral figure or polygon by an indefinite number; a line joining the extremities of two sides of a polygon, without itself being a side, is called a *diagonal* (AC). A polygon

is called *convex*, when the prolongations of the sides fall outside the polygon; in the opposite case it is said to be *not convex*.

7. When a straight line of fixed length (OA) revolves in a plane round one of its extremities, its other extremity describes a closed curved line called a *circle*; the fixed point O is called the centre and OA the *radius*; *all radii of the same circle are equal*; a line joining two points of the circumference is called a *chord* (BC); a *diameter* (DA) is a chord through the centre; *all diameters are equal*; a chord produced is called a *secant* (EF); it lies partly inside and partly outside the circle and cuts it in two points; if the secant be moved, so that the two points coincide in one point (I), this is called a *point of contact* and the line a *tangent* (GH); the tangent has only one point in common with the circle and lies wholly outside of it. A part of the circumference is called an *arc* ($\smile AC$). A part of the circle contained by a chord and an arc (BIC) is called a *segment*; a part contained by two radii and an arc (BOC) is called a *sector*.



A figure is said to be *inscribed* in a circle, when its sides are chords, *circumscribed*, when they are tangents.

8. Two figures are said to be *congruent*, when they are the same in everything, but only lying in different places; two congruent figures may be supposed to be placed on each other, so that they cover one another; the parts which then coincide are called *corresponding*. The sign for congruence is $\cong$. Circles with equal radii are congruent.

9. That which is given in a proposition is called *Hypothesis*, that which is to be proved, *Thesis*.

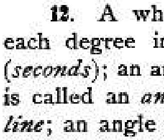
10. *Plane geometry* only treats of figures in the same plane, chiefly the straight line and the circle; Stereometry treats of lines, superficies, and bodies in general.

I.

THE SITUATION OF STRAIGHT LINES.

I. THE MUTUAL DEPENDENCE OF ANGLES.

11. When a straight line revolves round one of its points, until it again arrives at the position from whence it started, it is said to have completed a whole revolution; if it has not revolved so much, its position is determined by stating through how great a part of the revolution it has turned. We say that the line forms a certain *angle* with its former position, and *the angle between two lines is thus that part of a revolution, which one line must perform in order to cover the other.* The two lines are called the *legs* or *sides* of the angle, their point of intersection its *vertex*. The symbol for angle is $\angle$; where it cannot be misunderstood, an angle is denoted by a single letter at the vertex ($\angle x$ or $\angle A$); otherwise by three letters, one at the vertex and one at each of the legs, the first in the middle ($\angle BAC$).

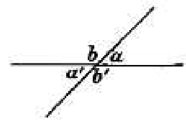


12. A whole revolution is divided into 360° (*degrees*), each degree into 60′ (*minutes*), and each minute into 60″ (*seconds*); an angle of 180°, corresponding to half a revolution, is called an *angle of continuation*; *its legs lie in a straight line*; an angle of 90°, corresponding to $\frac{1}{4}$ revolution, is called a *right angle* (symbol R.); angles less than 90° are called *acute*, between 90° and 180° *obtuse*. Angles which are not right angles are called *oblique*. Two lines which form a right angle are said to be at *right angles* or *perpendicular* to one another. The symbol for „perpendicular“ is $\perp$.

13. One angle is said to be the *complement* of another angle, when their sum is equal to 1R., and the *supplement*, when their sum equals 2R. Thus, when an angle is a°, its complement is $90^\circ - a^\circ$, its supplement $180^\circ - a^\circ$.

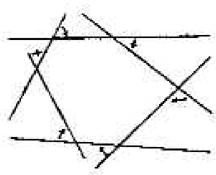
Two angles must therefore be equal, when their complements or supplements are equal.

14. *Adjacent angles* are those which have one leg in common and whose other legs are continuations of each other (b and a); two such angles are together equal to 180°, and are therefore supplementary angles.



15. *Vertical angles* are those whose legs are continuations of each other; *they are equal*, as the revolution which makes the legs of the one angle coincide also makes the legs of its vertical angle coincide; the angles formed by the intersection of two lines can therefore be found, when the one is known; for example, if $a = 45^\circ$, then also $a' = 45^\circ$, $b = b' = 135^\circ$.

16. *The sum of the exterior angles of a polygon is 4R.* For if a line be imagined to be placed on one of the sides of the polygon, from thence turned on to the next side and so on, till it again falls on the first side, it will by degrees have turned through all the exterior angles of the polygon; but it has thereby performed a whole revolution, that is 4R.*).



17. *The sum of the angles of a polygon is found by taking as many times 2R., as the polygon has sides, and from that subtracting 4R.* ($2n$R. — 4R., when it has n sides). For the sum of two adjacent angles is 2R.; therefore the angles

*) If some angular points of the polygon turn inwards, the proofs in 16 and 17 still hold good, when the revolution in backward direction is reckoned as negative.

We say in 16 that the line has performed a whole revolution, although it has arrived back on itself in a different way than in 11; we really hereby supplement the definition of the plane; the same reasoning could, for example, not be applied to arcs lying on the surface of a sphere.