SOLUTIONS OF EXAMPLES IN CONIC SECTIONS, TREATED GEOMETRICALLY

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Solutions of Examples in Conic Sections, Treated Geometrically by W. H. Besant

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IN

CONIC SECTIONS,

TREATED GEOMETRICALLY

BY

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THIRD EDITION, REVISED.

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PREFACE TO SOLUTIONS.

I HAVE frequently received requests for a book of Solutions of the Examples in my treatise on Conic Sections, but have never been able to find time to prepare them.

Mr Archer Green, B.A., Scholar of Christ's College, volunteered to undertake the task, with the aid of my notes and his own, and, with the exception of a few at the end, wrote out the solutions entirely.

Mr Green was however prevented by illness from completing the revision of the proofs, and I am much indebted to Mr J. Greaves, Fellow of Christ's College, who kindly undertook to examine the rest of the sheets.

The book will, I hope, prove useful both to students and teachers, as a companion volume to the treatise on Conic Sections.

W. H. BESANT.

Sept. 1881.

PREFACE TO THE THIRD EDITION.

THE solutions have been revised, and many additions have been made to them. They will now be found to be in complete accordance with the sixth edition of the Geometrical Conics.

W. H. BESANT.

Jan. 1890.

CONIC SECTIONS.

SOLUTIONS OF EXAMPLES.

CHAPTER I.

1. IF the tangent at P meet the directrix in Z, and S be the focus, PSZ is a right angle;

$\therefore S$ lies on the circle of which PZ is diameter.

2. Let PN and QM be the ordinates at P and Q.

Then $$PN : QM :: SP : SQ :: XN : XM;$$

$\therefore$ the triangles PXN and QXM are similar and PX, QX equally inclined to XS.

3. By Art. 8, FS is the external bisector of the angle PSQ.

4. $$SP : PK :: SA : AX :: SE : EK;$$

$$\therefore EP \text{ bisects the angle } SPK.$$

5. Since F, S, P and K lie on a circle,

the angle KSF = the angle FPK = the angle FTS.

6. $$PN : P'N' :: SP : SP';$$

$$\therefore XK : XN :: XK' : XN';$$

$\therefore$ the angle LNN' = the angle $K'N'X$ = the angle $LN'N$.

7. Let Q be the point where the tangent at R meets NP.

Then $NQ : NX :: SR : SX :: SA : AX :: SP : NX$;

$$\therefore SP = QN.$$

8. Let SY be perpendicular to the tangent at P and GL perpendicular to SP.

Then, since the triangles PSY, GPL are similar,

$$PG : PL :: SP : SY,$$

or

$$PG : SR :: SP : SY.$$

9. If the tangent meet the directrix in Z, and SP be drawn such that ZSP is a right angle meeting the tangent in P,

then P will be the point of contact of the tangent ZP.

10. If P, Q be the extremities of the chord, and PK, QL be perpendicular to the directrix,

$$SP : PK :: SA : AX :: SQ : QL;$$

$$\therefore SP + SQ :: PK + QL :: SA : AX.$$

Now the distance of the middle point of PQ from the directrix is equal to half $PK + QL$, and is therefore least when $SP + SQ$ is least, that is, when PQ goes through the focus.

11. If TP, TP' be the fixed tangents, and the tangent at Q meet them in E, E',

the angle $PSE =$ the angle ESQ, and the angle $QSE' =$ the angle $E'SP'$;

$$\therefore \text{the angle } ESE' = \text{half the angle } PSP'.$$

12. If perpendiculars from the given points PK, QL be drawn to the directrix and S be the focus,

$$SP : SQ :: PK : QL, \text{ a constant ratio};$$

$$\therefore \text{the locus of } S \text{ is a circle.}$$

13. Let the normal at P meet the axis in G. Taking O as the fixed point in the axis, it is obvious that the triangles OSR, GSP are similar;

$$\therefore SO : SR :: SG : SP :: SA : AX;$$

$\therefore$ SR is constant, and R lies on a circle of which S is the centre.

14. $$AT : AX :: SR : SX :: SA : AX;$$
$$\therefore AT = AS.$$

15. ST bisects the angle between SP and SQ, Art. 12, and SR bisects the angle between QS, and SP produced, Prop. II., Art. 5;

$$\therefore RST \text{ is a right angle.}$$

16. The triangles EAT, ERS are similar;

$$\therefore AT : SR :: EA : ER :: AX : SX;$$
$$\therefore AT : AX :: SR : SX :: SA : AX;$$
$$\therefore AT = AS.$$

17. If TL be perpendicular to the directrix,

$$SR : TL :: SA : AX :: SM : TL;$$
$$\therefore SM = SR.$$

18. FS is the external bisector of the angle QSP, and $F'S$ of QSP';

$$\therefore \text{the angle } FSF' = \text{half the angle } PSP'.$$

19. Since the triangles SPN, SGL are similar,

$$\therefore GL : PN :: SG : SP :: SA : AX.$$

20. If the normals PG, $P'G'$ meet in Q, and QV be drawn parallel to the axis to meet the chord in V,

$$VQ : VP :: SG : SP :: SA : AX :: SG' : SP' :: VQ : VP';$$
$$\therefore VP = VP', \text{ or } V \text{ bisects } PP'.$$