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THE EVOLUTION OF A GRAVITATING, ROTATING, CONDENSING FLUID

121

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In the science of Astronomy there are many observed phenomena for which, it is frankly admitted, there are at present no adequate explanations, while for other phenomena the physical causes assigned to explain them may legitimately be called into question. Certain of these phenomena are intimately associated with the evolution of matter in great masses as in a star or in the sun. Important mechanical changes with which observed phenomena are intimately associated must continually be taking place within a star as a result of the enormous amount of energy it is radiating into space.

Various phases of the mechanical problem have been discussed by mathematicians, among whom may be mentioned Harzer[1], Wilsing[2], Samson[3], Wilczynski[4] and Emden[5]. Their conclusions differ in some important particulars and they do not adequately explain observed phenomena. A further consideration of the problem seems not only desirable but to be an imperative need of astronomy at the present time.

Maclaurin, Jacobi, Thomson, Poincaré and Darwin have discussed the problem of figures of equilibrium of rotating homogeneous fluids and their stabilities. They take no account of probable differences of density in masses of great size nor of differences in angular velocities as observed in the sun. Professor Moulton[6] summarizes their work as follows:

"In particular, considering a series of homogeneous fluid masses of the *same* density but of *different* rates of rotation, it is shown that there is a continuous series of figures of stable equilibrium beginning with the sphere for zero rate of rotation; then, with increasing rotation, passing along a line of oblate spheroids until a certain rate of rotation is reached; then, with decreasing rate of rotation but with increasing moment of momentum, branching to a series of ellipsoids with three unequal axes, and continuing until a certain elongation is reached; and finally, at this point, branching to a series of so-called pear-shaped figures. It has been conjectured that if it were possible to follow the pear-shaped figures sufficiently far, it would be found that they would eventually reach a point where they would separate into two distinct masses.

"From this line of reasoning it has been regarded as probable that celestial masses, through loss of heat and consequent contraction, do break up in this way often enough to make the process an important one in cosmogony."

It is a commonly accepted theory that the sun and planets once existed as nebulae and that they, as well as the stars, developed from matter in a state of wide distribution. The problem presented by them in this condition is that of separate discrete particles possessing individual velocities and subject to accelerations produced by the force of gravitation due to their mutual attractions. It is essentially the problem of the orbital motion of n bodies, and as such must conform to the laws expressed by the general equations deduced for such motion.

I shall therefore regard a star, or the sun, as presenting the n body problem in which motions are modified and energy produced by the impacts to which the n bodies are subject.

In a general discussion of the problem, it would be necessary to consider masses of all sizes, such as might be encountered in nebulae, but when the configuration of particles has attained to the state of a star or the sun the n bodies with which we have to deal, will be the smallest divisions of matter,—namely, molecules. Whatever the masses composing the sun may have been when it was in a state of wide distribution, continued impact would tend to divide them into the smallest divisions of matter which can exist by themselves.

The motion of the center of mass may be considered equal to zero since in the present problem we are concerned only with the evolution of n bodies with respect to it. We are thus led to an immediate consideration of the three integrals of area which are expressed by the following equations[7]:

$$\begin{aligned} \sum_{i=1}^{n} m_i\left(x_i \frac{dy_i}{dt} - y_i \frac{dx_i}{dt}\right) &= C_z \\ \sum_{i=1}^{n} m_i\left(y_i \frac{dz_i}{dt} - z_i \frac{dy_i}{dt}\right) &= C_x \qquad (1) \\ \sum_{i=1}^{n} m_i\left(z_i \frac{dx_i}{dt} - x_i \frac{dz_i}{dt}\right) &= C_y. \end{aligned}$$

An areal velocity is essentially a vector quantity and therefore not only its magnitude but also its direction must be specified. An arrow in the axis of rotation of length equal to the magnitude of the areal velocity is its conventional vector representation, and its origin is usually in the plane of rotation to which it is always perpendicular. Its direction of pointing indicates the direction of rotation, since it is drawn with reference to this rotation in such a way that the arrow points in the direction a right-handed screw would advance if turned in the direction of rotation.

Let the areal velocity of particle i be represented by a vector A_i, its mass by m_i, and the product m_iA_i by the vector C_i which we shall call its mass rotation.

A vector representation of mass rotations in the problem of n bodies would consist of n arrows of lengths equal to the mass rotations of the respective particles, having the center of mass as common origin, and pointing outward in directions corresponding to the rotations which they were drawn to represent. Let each of the n vectors thus drawn be resolved into components and their sums be taken along the three coördinate axes, those components which point in the positive direction of the axes being considered positive; those pointing in the opposite direction negative. When this is done the geometrical significance of the constants C_x, C_y and C_z in equation (1) is at once apparent. They are the respective sums of these components along the respective axes. Thus C_x expanded takes the form

$$C_x = C_{x1} + C_{x2} + (-C_{x3}) + (-C_{x4}) + \cdots + C_{xn} = \sum_{i=1}^{n} C_{xi} \qquad (2)$$

with similar expressions for C_y and C_z.

Take the vector sum of all the vectors representing mass rotations and we determine in direction and amount the excess of rotation of the configuration and the plane in which it takes place. This condition is conveniently expressed by the vector equation[8],

$$\sum_{i=1}^{n} m_i \bar{r}_i \times r_i \frac{\overline{d\theta_i}}{dt} = \sum_{i=1}^{n} m_i \bar{A}_i = \sum_{i=1}^{n} \bar{C}_i \qquad (3)$$