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SYNTHETIC SOLID GEOMETRY

BY

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PREFACE.

THE matter of the present work has, with some varia-
tions, been in manuscript for a number of years, and has
formed the subject of an annual course of lectures to
mathematical students by whom the subject has been
well received as one of the most interesting in the earlier
part of a mathematical course.

I have been induced to present the work to the public,
partly, by receiving from a number of Educationists
inquiries as to what work on Solid Geometry I would
recommend as a sequel to my Plane Geometry, and partly,
from the high estimate that I have formed of the value
of the study of synthetic solid geometry as a means of
mental discipline.

To me it seems to exercise not only the purely intel-
lectual powers in the development of its theorems, but
also the imagination in the mental building-up of the
necessary spatial figures, and the eye and the hand in
their representations.

In this work the subject is carried somewhat farther
than is customary in those works in which the subject
of solid geometry is appended to that of plane geometry,

but the extensions thus made are fairly within the scope of an elementary work, and are highly interesting and important in themselves as forming valuable aids to the right understanding of the more transcendental methods.

It appears to me that it is a prevalent custom to lay too little stress on synthetic methods as soon as plane geometry is passed, and to hurry the student too rapidly into the analytic methods. If mathematical knowledge is all that is required, this may possibly be an advantageous course; but if mental culture is, as it should be, the chief end in a university education, this customary usage is not the best one.

I have found it convenient to divide the work into four parts, each of which is further divided into sections.

The first part deals with a consideration of the descriptive properties of lines and planes in space, of the polyhedra, and of the cone, the cylinder, and the sphere.

Here I would feel like apologising for the introduction of a new term, were it not that I believe that its introduction will be fully justified by a careful perusal of the work.

Legendre, in his notes to his geometry, proposed to use the word 'corner' (coin) for the figure formed by the meeting of two planes, and he considered that the different polyhedral angles should receive special names as being geometrical figures of different species. Without

discussing this idea, I have employed the word 'corner' to denote a solid or polyhedral angle of not less than three faces, while I have retained the expression 'dihedral angle' in its usual sense. If a dihedral angle be cut by a plane, this cutting plane necessarily cuts through both faces, and the figure of intersection is a plane angle. Whereas, if any polyhedral angle be cut by a plane which intersects all its faces, the figure of section is not a simple angle, but a polygon. Thus the plane angle and the dihedral have this in common, that they can both be measured by the same kind of angular unit, while the affinities of the polyhedral angle are with the polygon.

Moreover, the trihedral angle is a geometrical function of three plane angles and three dihedral angles, neither of which exists without the other, and every polyhedral angle is a geometrical function or combination of plane and dihedral angles, and these form its elements. Hence I have used the term 'three-faced corner' for 'trihedral angle,' and generally 'n-faced corner' for 'n-hedral angle.' This nomenclature is very convenient; but if any Teacher prefers the older forms, he can readily make the necessary change in language.

The rectangular parallelepiped should certainly be supplied with some convenient name. I have adopted the term 'cuboid,' as proposed by Mr. Hayward, as being both convenient and suggestive.

The second part of the work deals with areal rela-
tions, that is, the relations among the areas of squares
and rectangles on characteristic line-segments of the
prominent spatial figures.

The majority of the results, besides being highly inter-
esting in themselves, form data for subsequent higher
work.

The third part is devoted to stereometry and planimetry.
In this are developed the principal rules and formulæ
for the measurement of volumes and surfaces of the
more prominent spatial figures which admit of such
measurement, and a special section is given to the con-
sideration of volumes and surfaces generated by moving
areas and lines, and to the development of the theorems
of Pappus or Guldinus.

The fourth and last part begins with an explanation
of the principles of conical or perspective projection.
By the application of these principles in projecting a
circle into a cone and cutting the cone by a plane, the
student is introduced to the conic, and is led to under-
stand its meaning, and the relations of the various conics
to one another.

The more common properties of the conics are then
easily obtained through a study of the curve as a plane
section of a circular cone. The latter half of this part
is given to spheric geometry. The spheric figure (tri-
angle and polygon) is considered as the section of a