

**AN ELEMENTARY TREATISE ON
THE LUNAR THEORY: WITH A
BRIEF SKETCH OF THE HISTORY
OF THE PROBLEM UP TO THE TIME
OF NEWTON**

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An Elementary Treatise on the Lunar Theory: With a Brief Sketch of the History of the Problem up to the Time of Newton by Hugh Godfray

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HUGH GODFRAY

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WITH
A BRIEF SKETCH OF THE HISTORY OF THE PROBLEM
UP TO THE TIME OF NEWTON.

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PREFACE TO THE FIRST EDITION.

OF all the celestial bodies whose motions have formed the subject of the investigations of astronomers, the Moon has always been regarded as that which presents the greatest difficulties, on account of the number of inequalities to which it is subject; but the frequent and important applications of the results render the Lunar Problem one of the highest interest, and we find that it has occupied the attention of the most celebrated astronomers from the earliest times.

Newton's discovery of Universal Gravitation, suggested, it is supposed, by a rough consideration of the motions of the moon, led him naturally to examine its application to a more severe explanation of her disturbances; and his Eleventh Section is the first attempt at a theoretical investigation of the Lunar inequalities. The results he obtained were found to agree very nearly with those determined by observation, and afforded a remarkable confirmation of the truth of his great principle; but the geometrical methods which he had adopted seem inadequate to so complicated a theory, and recourse has been had to analysis for a complete determination of the disturbances, and for a knowledge of the true orbit.

The following pages will, it is hoped, form a proper introduction to more recondite works on the subject: the difficulties which a person entering upon this study is most likely to stumble at, have been dwelt upon at considerable length, and though different methods of investigation have been employed by different astronomers, the difficulties met with are nearly the same, and the principle of successive approximation is common to all. In the present work, the approximation is carried to the second order of small quantities, and this, though far from giving accurate values, is amply sufficient for the elucidation of the method.

The differences in the analytical solutions arise from the various ways in which the position of the moon may be indicated by altering the system of coordinates to which it is referred, or again, in the same system, by choosing different quantities for independent variables.

D'Alembert and Clairaut chose for coordinates the projection of the radius vector on the plane of the ecliptic and the longitude of this projection. To form the differential equations, the true longitude was taken for independent variable.

To determine the latitude, they, by analogy to Newton's method, employed the differential variations of the motion of the node and of the inclination of the orbit.

Laplace, Damoiseau, Plana, and also Herschel and Airy in their more elementary works, have found it more convenient to express the variations of the latitude directly, by an equation of the same form as that of the radius vector.

Lubbock and Pontécoulant, taking the same coordinates of the moon's position, make the time the independent variable; and when it is desired to carry the approximation to a high order, this method offers the advantage of not requiring any reversion of series.

Poisson proposed the method used in the planetary theory, that is, to determine the variation in the elements of the moon's orbit, and thence to conclude the corresponding variations of the radius vector, the longitude, and the latitude.

The selection of the method followed in the present work, which is the same as that of Airy, Herschel, &c., was made on account of its simplicity: moreover, it is the method which has obtained in this university, and it is hoped that it may prove of service to the student in his reading for the examination for Honours. In furtherance of this object, one of the chapters (the sixth) contains the physical interpretation of the various important terms in the radius vector, latitude, and longitude.*

The seventh chapter, or Appendix, contains some of the most interesting results in the terms of the higher orders, among which will be found the values of c and g completely obtained to the third order.

The last chapter is a brief historical sketch of the Lunar Problem up to the time of Newton, containing an account of the discoveries of the several inequalities and of the methods by which they were represented, those only being

* See the Report of the "Board of Mathematical Studies" for 1850.

mentioned which, as the theory has since verified, were real onward steps. The perusal of this chapter will shew to what extent we are indebted to our great philosopher; at the same time we cannot fail being impressed with reverence for the genius and perseverance of the men who preceded him, and whose elaborate and multiplied hypotheses were in some measure necessary to the discovery of his simple and single law.

I take this opportunity of acknowledging my obligations to several friends, whose valuable suggestions have added to the utility of the work.

HUGH GODFRAY.

*St. John's College, Cambridge,
April, 1858.*

IN the present edition, besides the change of form and the incorporation of the figures with the text, which it is hoped will render the work more commodious, very few alterations have been thought necessary; and, except in one or two instances, where additional paragraphs have been introduced, nothing but the wording of some of the sentences has been altered.

October, 1859.

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