

# **ELEMENTARY CALCULUS**

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Elementary calculus by William F. Osgood

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**WILLIAM F. OSGOOD**

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# ELEMENTARY CALCULUS

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## PREFACE

THE object of this book is to present the elements of the Differential Calculus in a form easily accessible for the undergraduate. It is possible, from the very beginning, to illustrate the ideas and methods of the Calculus by means of applications to physics and geometry, which the student can readily grasp, and which will seem to him of interest and value. To do this, the stress in the illustrative examples worked in the text must be laid first of all on the thought which underlies the method of solution, in distinction from the exposition of a process, reduced in the worst teaching to rules, whereby the answer can be obtained. The treatment of maxima and minima, Chapter III, §§ 2, 3, and curve tracing, Chapter III, § 5 and Chapter VII, § 10, will serve to show what is here meant.

It is, however, also essential that the student receive thorough training in the formal processes and the technique of the Calculus, and this side has been treated with care and completeness. Note, for example, the differentiation of composite functions in Chapter II, § 8, and the exposition of the use of differentials in differentiating in Chapter IV, §§ 4, 5.

An important application of the graphical methods, with which the Calculus is so intimately associated, is that of solving approximately numerical equations which do not come under the standard rules of algebra and trigonometry. Hitherto, however, little attempt has been made to present this subject, simple as it is, in any systematic and elementary manner. In Chapter VII the common methods in use by physicists and others who apply the Calculus are set forth and illustrated by simple examples.

The book might have included a brief treatment of curvature and evolutes, and the cycloid. But probably most

teachers of the Calculus will prefer to take up integration next, and so the closing chapter is devoted to the last of the elementary functions, the inverse trigonometric functions, with special reference to their one great application in the elements of mathematics, namely, their application to integration.

The book is so written that it can be adapted, if desired, to an abridged course, in which, after the fundamentals of the first three chapters have been covered, any of the remaining topics can be treated briefly, and thus a wide scope in subject matter is possible, even when the time is short.

CAMBRIDGE, MASSACHUSETTS,  
January, 1921.

# CONTENTS

## CHAPTER I

### INTRODUCTION

	PAGE
1. Functions . . . . .	1
2. Continuation. General Definition of a Function . . . . .	10

## CHAPTER II

### DIFFERENTIATION OF ALGEBRAIC FUNCTIONS. GENERAL THEOREMS

1. Definition of the Derivative . . . . .	13
2. Differentiation of $x^n$ . . . . .	16
3. Differentiation of a Constant . . . . .	20
4. Differentiation of $\sqrt{x}$ . . . . .	21
5. Three Theorems about Limits. Infinity . . . . .	22
6. General Formulas of Differentiation . . . . .	29
7. General Formulas of Differentiation, Continued . . . . .	32
8. General Formulas of Differentiation, Concluded . . . . .	35
9. Differentiation of Implicit Algebraic Functions . . . . .	39

## CHAPTER III

### APPLICATIONS

1. Tangents and Normals . . . . .	46
2. Maxima and Minima . . . . .	49
3. Continuation : Auxiliary Variables . . . . .	53
4. Increasing and Decreasing Functions . . . . .	60
5. Curve Tracing . . . . .	64
6. Relative Maxima and Minima. Points of Inflection . . . . .	67
7. Necessary and Sufficient Conditions . . . . .	71
8. Velocity ; Rates . . . . .	72