

**ELEMENTS OF DYNAMIC: AN
INTRODUCTION TO THE STUDY
OF MOTION AND REST IN SOLID
AND FLUID BODIES; PART I.
KINEMATIC**

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W. K. CLIFFORD

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ELEMENTS OF DYNAMIC

AN INTRODUCTION TO THE STUDY OF

MOTION AND REST

IN SOLID AND FLUID BODIES

BY

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BOOK I. TRANSLATIONS.

CHAPTER I. STEPS.

INTRODUCTION.

JUST as Geometry teaches us about the *sizes* and *shapes* and *distances* of bodies, and about the relations which hold good between them, so Dynamic teaches us about the changes which take place in those distances, sizes, and shapes (which changes are called *motions*), the relations which hold good between different motions, and the circumstances under which motions take place.

Motions are generally very complicated. To fix the ideas, consider the case of a man sitting in one corner of a railway carriage, who gets up and moves to the opposite corner. He has gone from one place to another; he has turned round; he has continually changed in shape, and many of his muscles have changed in size during the process.

To avoid this complication we deal with the simplest motions first, and gradually go on to consider the more complex ones. In the first place we postpone the consideration of changes in size and shape by treating only of those motions in which there are no such changes. A body which does not change its size or shape during the time considered is called a *rigid* body.

The motion of rigid bodies is of two kinds; change of place, or *translation*, and change of direction or aspect, which is called *rotation*. In a motion of pure *translation*, every straight line in the body remains parallel to its original position; for if it did not, it would turn round,