AN ELEMENTARY TREATISE ON HYDRODYNAMICS AND SOUND

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An elementary treatise on hydrodynamics and sound by A. B. Basset

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BY

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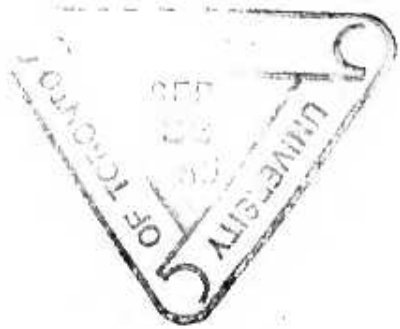

PREFACE.

THE treatise on Hydrodynamics, which I published in 1888, was intended for the use of those who are acquainted with the higher branches of mathematics, and its aim was to present to the reader as comprehensive an account of the whole subject as was possible. But although a somewhat formidable battery of mathematical artillery is indispensable to those who desire to possess an exhaustive knowledge of any branch of mathematical physics, yet there are a variety of interesting and important investigations, not only in Hydrodynamics, but also in Electricity and other physical subjects, which are well within the reach of every one, who possesses a knowledge of the elements of the Differential and Integral Calculus and the fundamental principles of Dynamics. I have accordingly, in the present work, abstained from introducing any of the more advanced methods of analysis, such as Spherical Harmonics, Elliptic Functions and the like; and, as regards the dynamical portion of the subject, I have endeavoured to solve the various problems which present themselves, by the aid of the Principles of Energy and Momentum, and have avoided the use of Lagrange's equations. There are a few problems, such as the helicoidal steady motion and stability of a solid of revolution moving in an infinite liquid, which cannot be conveniently treated without having recourse to moving axes; but as the theory of moving axes is not an altogether easy branch of Dynamics, I have as far as possible abstained from introducing them, and the reader who is unacquainted with the use of moving axes is recommended to omit those sections in which they are employed.

The present work is principally designed for those who are reading for Part I. of the Mathematical Tripos, under the new regulations, and for other examinations in which an elementary knowledge of Hydrodynamics and Sound is required; but I also trust that it will be of service, not only to those who have neither the time nor the inclination to become conversant with the intricacies of the higher mathematics, but that it will also prepare the

way for the acquisition of more elaborate knowledge, on the part of those who have an opportunity of devoting their attention to the more recondite portions of these subjects.

The first part, which relates to Hydrodynamics, has been taken with certain alterations and additions from my larger treatise, and the analytical treatment has been simplified as much as possible. I have thought it advisable to devote a chapter to the discussion of the motion of spheres and circular cylinders, in which the equations of motion are obtained by the direct method of calculating the resultant pressure exerted by the liquid upon the solid; inasmuch as this method is far more elementary, and does not necessitate the use of Green's Theorem, nor involve any further knowledge of Dynamics on the part of the reader, than the ordinary equations of motion of a rigid body. The methods of this chapter can also be employed to solve the analogous problem of determining the electrostatic potential of cylindrical and spherical conductors and accumulators, and the distribution of electricity upon such surfaces. The theory of the motion of a solid body and the surrounding liquid, regarded as a single dynamical system, is explained in Chapter III., and the motion of an elliptic cylinder in an infinite liquid, and the motion of a circular cylinder in a liquid bounded by a rigid plane, are discussed at length.

The Chapter on Waves and on Rectilinear Vortex Motion comprises the principal problems which admit of treatment by elementary methods, and I have also included an investigation due to Lord Rayleigh, respecting one of the simpler cases of the instability of fluid motion.

In the second part, which deals with the Theory of Sound, I have to acknowledge the great assistance which I have received from Lord Rayleigh's classical treatise. This part contains the solution of the simpler problems respecting the vibrations of strings, membranes, bars and gases; and I have also added a few pages on the statical problem of the flexion of bars. A few sections are also devoted to the Thermodynamics of perfect gases, principally for the sake of supplementing Maxwell's treatise on Heat, by giving a proof of some results which require the use of the Differential Calculus.

I have to express my best thanks to Professor Greenhill for having read the proof sheets, and for having made many valuable suggestions during the progress of the work.

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