A TREATISE ON BESSEL FUNCTIONS AND THEIR APPLICATIONS TO PHYSICS

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A Treatise on Bessel Functions and Their Applications to Physics by Andrew Gray & G. B. Mathews

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ANDREW GRAY & G. B. MATHEWS

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"And as for the *Mixt Mathematikes* I may onely make this prediction, that there cannot faile to bee more kindes of them, as Nature growes further disclosed."

BACON.

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MACMILLAN AND CO.

AND NEW YORK.

1895

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PREFACE.

THIS book has been written in view of the great and growing importance of the Bessel functions in almost every branch of mathematical physics; and its principal object is to supply in a convenient form so much of the theory of the functions as is necessary for their practical application, and to illustrate their use by a selection of physical problems, worked out in some detail.

Some readers may be inclined to think that the earlier chapters contain a needless amount of tedious analysis; but it must be remembered that the properties of the Bessel functions are not without an interest of their own on purely mathematical grounds, and that they afford excellent illustrations of the more recent theory of differential equations, and of the theory of a complex variable. And even from the purely physical point of view it is impossible to say that an analytical formula is useless for practical purposes; it may be so *now*, but experience has repeatedly shown that the most abstract analysis may unexpectedly prove to be of the highest importance in mathematical physics. As a matter of fact it will be found that little, if any, of the analytical theory included in the present work has failed to be of some use or other in the later chapters; and we are so far from thinking that anything superfluous has been inserted, that we could almost wish that space would have allowed of a more extended treatment, especially in the chapters on the complex theory and on definite integrals.

With regard to that part of the book which deals with physical applications, our aim has been to avoid, on the one hand, waste of

time and space in the discussion of trivialities, and, on the other, any pretension of writing an elaborate physical treatise. We have endeavoured to choose problems of real importance which naturally require the use of the Bessel functions, and to treat them in considerable detail, so as to bring out clearly the direct physical significance of the analysis employed. One result of this course has been that the chapter on diffraction is proportionately rather long; but we hope that this section may attract more general attention in this country to the valuable and interesting results contained in Lommel's memoirs, from which the substance of that chapter is mainly derived.

It is with much pleasure that we acknowledge the help and encouragement we have received while composing this treatise. We are indebted to Lord Kelvin and Professor J. J. Thomson for permission to make free use of their researches on fluid motion and electrical oscillations respectively; to Professor A. Lodge for copies of the British Association tables from which our tables IV., V., VI., have been extracted; and to the Berlin Academy of Sciences and Dr Meissel for permission to reprint the tables of J_0 and J_1 which appeared in the *Abhandlungen* for 1888. Dr Meissel has also very generously placed at our disposal the materials for Tables II. and III., the former in manuscript; and Professor J. McMahon has very kindly communicated to us his formulæ for the roots of $J_n(x) = 0$ and other transcendental equations. Our thanks are also especially due to Mr G. A. Gibson, M.A., for his care in reading the proof sheets. Finally we wish to acknowledge our sense of the accuracy with which the text has been set up in type by the workmen of the Cambridge University Press.

The bibliographical list on pp. 289—291 must not be regarded as anything but a list of treatises and memoirs which have been consulted during the composition of this work.

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