

**MATHEMATICAL
MONOGRAPHS. NO.
3. DETERMINANTS**

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LAENAS GIFFORD WELD

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EDITED BY

MANSFIELD MERRIMAN AND ROBERT S. WOODWARD.

No. 3.

DETERMINANTS.

BY

LAENAS GIFFORD WELD,

PROFESSOR OF MATHEMATICS IN THE STATE UNIVERSITY OF IOWA.

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UNDER THE TITLE

HIGHER MATHEMATICS.

First Edition, September, 1896.

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ROBERT DRUMMOND, PRINTER, NEW YORK.

EDITORS' PREFACE.

THE volume called *Higher Mathematics*, the first edition of which was published in 1896, contained eleven chapters by eleven authors, each chapter being independent of the others, but all supposing the reader to have at least a mathematical training equivalent to that given in classical and engineering colleges. The publication of that volume is now discontinued and the chapters are issued in separate form. In these reissues it will generally be found that the monographs are enlarged by additional articles or appendices which either amplify the former presentation or record recent advances. This plan of publication has been arranged in order to meet the demand of teachers and the convenience of classes, but it is also thought that it may prove advantageous to readers in special lines of mathematical literature.

It is the intention of the publishers and editors to add other monographs to the series from time to time, if the call for the same seems to warrant it. Among the topics which are under consideration are those of elliptic functions, the theory of numbers, the group theory, the calculus of variations, and non-Euclidean geometry; possibly also monographs on branches of astronomy, mechanics, and mathematical physics may be included. It is the hope of the editors that this form of publication may tend to promote mathematical study and research over a wider field than that which the former volume has occupied.

December, 1905.

AUTHOR'S PREFACE.

THE author of the present volume feels some embarrassment in having already offered to the public a work upon the Theory of Determinants. The apparently general acceptability of this former work, which has now reached its third edition, doubtless led to his being invited by the editors of Higher Mathematics to prepare for them a chapter upon the same subject. This was done without the least thought of its publication as a separate volume. Now that its issue as such, along with the other chapters, is requested by both the publishers and the editors of Higher Mathematics, it is but just to the author that the above circumstances should be understood lest he be suspected of entertaining an unseemly desire to keep himself before the mathematical public by vain repetition.

The limitations imposed have permitted the addition of only a few articles to the work as originally published; principally those treating of linear substitutions, quantics, invariance, covariance, and functional determinants. Determinants of special forms have not been considered, nor is there the least reference to the application of determinants to geometry. It is hoped, however, that the work may prove useful to the constantly increasing number of students who, while not wishing to specialize in mathematics, desire to obtain the comprehensive view of its methods and processes essential to the successful pursuit of the exact sciences in general.

IOWA CITY, IOWA, U. S. A.,
December, 1905.

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DETERMINANTS.

ART. I. INTRODUCTION.

AS early as 1693 Leibnitz arrived at some vague notions regarding the functions which we now know as determinants. His researches in this subject, the first account of which is contained in his correspondence with De L'Hospital, resulted simply in the statement of some rather clumsy rules for eliminating the unknowns from systems of linear equations, and exerted no influence whatever upon subsequent investigations in the same direction. It was over half a century later, in 1750, that Gabriel Cramer first formulated an intelligible and general definition of the functions, based upon the recognition of the two classes of permutations, as presently to be set forth.

Though Cramer failed to recognize, even to the same extent as Leibnitz, the importance of the functions thus defined, the development of the subject from this time on has been almost continuous and often rapid. The name "determinant" is due to Gauss, who, with Vandermonde, Lagrange, Cauchy, Jacobi, and others, ranks among the great pioneers in this development.

Within recent years the theory of determinants has come into very general use, and has, in the hands of such mathematicians as Cayley and Sylvester, led to results of the greatest interest and importance, both through the study of special forms of the functions themselves and through their applications.*

* A list of writings on Determinants is given by Muir in *Quarterly Journal of Mathematics*, 1881, Vol. XVIII, pp. 110-149.