KEY TO OLNEY'S ELEMENTS OF GEOMETRY AND PLANE AND SPHERICAL TRIGONOMETRY

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TO

OLNEY'S ELEMENTS OF GEOMETRY

AND

PLANE AND SPHERICAL TRIGONOMETRY,

WITH

AN INTRODUCTION ON METHOD OF TEACHING.

SOLUTIONS BY

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1880.

TEACHING GEOMETRY.

As the text-book to which this volume is a key differs largely in its spirit and plan from the older geometries, it has been thought well that the author should offer a few suggestions as to the best method of using it. In doing this, he trusts that long experience and observation as to methods and results will be deemed a sufficient apology for the broader scope which these suggestions will be seen to take; i. e., suggestions as to methods of teaching the subject, irrespective of the particular text-book used. By permission, the author refers to the articles on Pure Mathematics, and especially to the article on Geometry, in that teacher's thesaurus, "The Cyclopædia of Education," by Kiddle and Schem, as setting forth — more at length than can be done in this place — the author's views on these subjects.

As to Part I. of the Geometry.

The first thing that will strike the teacher upon opening this Geometry is the introductory portion of the work, called *Part First*. This part presents the principal definitions and facts of Plane Geometry, with ample illustrations of their meaning and practical nature. There is no attempt at demonstration. This course has been taken in obedience to the canon of the teacher's art which prescribes "facts before theories," and that the perceptive faculties must supply the facts upon which the logical faculty reasons. A large part of all the difficulty which pupils encounter in

the study of geometry grows out of a want of a clear and abiding impression of the subjects of thought and the aims of the science. Geometry is a double science; on one hand it is pure logic, and on the other it is a system of practical truths concerning the properties of form. One may understand thoroughly the logic which demonstrates that the tangent is a mean between the secant and its external segment, and yet know nothing of the practical significance and utility of the fact. Indeed, he may not even apprehend the statement at all as a practical fact of magnitude and form.

As this is a vital point in teaching geometry, let us illustrate it. Suppose a class of a dozen ordinary pupils have come in the old way to this proposition, and have learned to demonstrate it satisfactorily. Then let the teacher draw a square on the tangent, and ask who can draw an oblong of the same size. Not half — perhaps not one — of the class can do it, and many will not even know what is meant. Allow an illustration from the author's experience. During the past twenty years he has examined scores of young men, — candidates for advanced standing in college, — who had been over geometry in the ordinary way. Whenever he has found one who showed special attainment, he has asked something like the following series of questions: —

How do you find the area of a spherical triangle?

What is a tri-rectangular triangle? What part of the surface of a sphere?

What is spherical excess?

Can you demonstrate the truths declared?

In most cases the questions have been answered promptly and correctly, and one or more demonstrations have been given.

A practical problem has then been given: Here is a sphere two feet in diameter; that spherical triangle on its surface has its angles 95°, 80°, and 110°. What is its area?

No student of all the number has been found who could obtain the answer!

It is such reflections and such facts as these, confirmed by the views of such eminent educators as ex-President Hill of Harvard, and the concurrent sentiment of writers on pedagogy, that have given Part I. its character and form.

That the method of giving some acquaintance with the practical truths of geometry, before attempting to demonstrate them, works well in practice, has been demonstrated again and again. Near twenty years ago, the author was teaching in a school where both sexes were in his classes. Conic sections, taught by the old method, never failed to be a drudgery to the girls, and to all but a few of the boys. At length he hit upon this plan: The first few lessons were occupied in teaching how to draw the conic sections neatly and accurately; how to draw a tangent to each; how to find the centre, axes, foci, &c. In fact, all the leading properties were taught *as facts;* and the pupils were trained to draw, with great neatness and elegance, the figures illustrating or applying them. In this way girls and all became enthusiastic, and "Why is it so?" became an oft-repeated and eager question. After this introduction, the terror and drudgery of the science disappeared. They knew what the truths were, and were interested in them as facts, and had an eager desire to know why they were so.

This, the author believes, will inevitably be the result of teaching geometry in the method proposed; indeed, he knows it may be so, from the large number of testimonies he has received from teachers *who have tried the method in the spirit of it.*

WHAT, THEN, IS THE METHOD? — It is pretty fully indicated in the book itself. It would be well if Part I. were introduced into the course in what is usually called the "grammar grade" of our public schools, as soon as an elementary knowledge of arithmetic is secured, and before that subject is completed, or algebra is commenced.

Pupils of this age memorize readily, and are fond of drawing. "The elements of geometry are much easier to learn, and are of more value when learned, than advanced arithmetic; and, if a boy is to leave school with merely a grammar-school education, he would be better prepared for the active duties of life with a *little* arithmetic and *some* geometry, than with *more* arithmetic and *no* geometry." — MARKS. "It will be found that children of this age are quicker at comprehending first lessons in geometry than those of fifteen. The child's powers of sensation are developed before his powers of conception, and these before his reasoning powers. This is, therefore, the true order of education; and a powerful logical drill, like mental arithmetic, is sadly out of place in the hands of a child whose powers of observation and conception have, as yet, received no training whatever." — EX-PRESIDENT HILL *of Harvard.*

That something of this kind is needed in our lower schools, is apparent in the tendency to introduce lessons on form, drawing, &c. But why not make these lessons such as to lay a good foundation for subsequent study? Let Part I. occupy about half of the third year of this grade, and it will be found to be pursued with delight, and will greatly facilitate the subsequent study of geometry. But, if the study of this part is not introduced here, it should not be omitted or slighted when the study of geometry proper is commenced a few years later. A high-school or preparatory class commencing the study of geometry at the ordinary age will save time, and also secure a more intelligent interest in the subject, by spending six weeks on this part.

The points to be secured are: —

1. To learn, thoroughly and *intelligently*, all the definitions.

These should be committed to memory, and repeated until they can be recalled without conscious effort; great pains being taken that the exact language of the text is used, unless the teacher substitutes other language for reason.

Of course we do not mean that this shall be a mere *memoriter* exercise. The greatest pains should be taken to secure a thorough understanding of the technical terms, and of the language of the definitions. For this purpose the illustrations in the text will afford helps to the pupil and hints to the teacher.

A pupil should always be able, not only to recite the definition readily and accurately, but also to illustrate its meaning in the manner of the book, in his own language, and by a figure, when that is possible.

Clearness of perception, and readiness and accuracy of expression, is the motto in doing this work.

2. The second line of effort should be to train the pupil to solve every problem with neatness and despatch. Much practice in drawing smooth lines *in ink*, with a ruling-pen, will be necessary in order that the pupils shall be able to execute the figures properly.

An excellent plan is to have each pupil make a portfolio of drawings with the utmost care on separate sheets of drawing-paper. This should contain every exercise of Part I. A portfolio of separate sheets is better than a book in which the pages are fastened, since no drawings need be put into it till they are satisfactory. A little care at the outset will make this a most pleasing and profitable course of study.

In this part of the work, never allow a pupil to rest satisfied with being able to tell how a thing is to be done, but require that he actually do it with ruler and dividers. These two instruments, with a ruling-pen, are the sole ones needed, or which should be allowed in this work.

When working in class, on the blackboard, a string will take the place of dividers. Never allow a pupil to guess at a construction, at this stage of his course. If a line is said to be perpendicular to another, let it be made so on geometrical principles. If an angle is said to be bisected, let the geometrical construction be made.

This *punctilio* in doing all on correct principles will secure several very important results: 1st, It will fix permanently in mind all the working facts of plane geometry. 2d, It will train the eye to the perception of form. After a course of training like this, a pupil when making a freehand drawing (as is sometimes allowable and even desirable), in subsequent work, will not make an angle of 80° for a right angle, or draw any sort of a triangle when his problem requires an equilateral triangle. His trained geometric sense will not allow this. 3d, This habit of *seeing* the import of geometrical language will give a clearness of apprehension, and a knowledge of relations, which will be of vast importance in subsequent work. 4th, It will give him the *geometrical spirit*, i.e., the instinct, if we may so call it, that leads him to investigate by constructions, and by scrutinizing relations as they appear in geometrical figures.

As to Part II.

The first thing that will strike one here is the division of the subject into chapters and sections, instead of books, as has been the prevailing custom since the time of Euclid (about 300 or 400 B.C). The author's reasons for this are the same as have led writers on most other subjects to abandon the ancient method of division into books, for the modern one of division into chapters and sections. These are, mainly, two: viz., first, the word "book" has become nearly obsolete in the former sense; and, second, the division into chapters, and the subdivision into sections, furnishes a better opportunity for an analysis of the subject. Elementary geometry is naturally divided by its subject-matter into plane and solid; i. e., into the treatment of forms and magnitudes confined to one plane, and of those not so restricted. We thus need, at the outset, a more comprehensive division than the old "book," in order to mark the natural division of the subject. Again, these parts need subdividing, in order to