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ELLIPTIC INTEGRALS

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CONTENTS

INTRODUCTION

THE editors of the present series of mathematical monographs have requested me to write a work on elliptic integrals which "shall relate almost entirely to the three well-known elliptic integrals, with tables and examples showing practical applications, and which shall fill about one hundred octavo pages." In complying with their request, I shall limit the monograph to what is known as the Legendre-Jacobi theory; and to keep the work within the desired number of pages I must confine the discussion almost entirely to what is known as the elliptic integrals of the first and second kinds.

In the elementary calculus are found methods of integrating any rational expression involving under a square root sign a quadratic in one variable; in the present work, which may be regarded as a somewhat more advanced calculus, we have to integrate similar expressions where cubics and quartics in one variable occur under the root sign. Whatever be the nature of these cubics and quartics, it will be seen that the integrals may be transformed into standard normal forms. Tables are given of these normal forms, so that the integral in question may be calculated to any degree of exactness required.

With the trigonometric sine function is associated its inverse function, an integral; and similarly with the normal forms of elliptic integrals there are associated elliptic functions. A short account is given of these functions which emphasizes their doubly periodic properties. By making suitable transformations and using the inverse of these functions, it is found that the integrals in question may be expressed more concisely through the normal forms and in a manner that indicates the transformation employed.

The underlying theory, the philosophy of the subject, I have attempted to give in my larger work on elliptic functions, Vol. I. In the preparation of the present monograph much use has been made of Greenhill's *Application of Elliptic Functions*, a work which cannot be commended too highly; one may also read with great advantage Cayley's *Elliptic Functions*. The standard works of Legendre, Abel and Jacobi are briefly considered in the text. It may also be of interest to note briefly the earlier mathematicians who made possible the writings just mentioned.

The difference of two arcs of an ellipse that do not overlap may be expressed through the difference of two lengths on a straight line; in other words, this difference may be expressed in an *algebraic* manner. This is the geometrical signification of a theorem due to an Italian mathematician, Fagnano, which theorem is published in the twenty-sixth volume of the *Giornale de' letterari d'Italia*, 1716, and later with numerous other mathematical papers in two volumes under the title *Produzioni mathematiche del Marchese Giulio Carlo de' Toschi di Fagnano*, 1750.

The great French mathematician Hermite (*Cours*, rédigé par Andoyer, Paris, 1882) writes "*Ce résultat doit être cité avec admiration comme ayant ouvert le premier la voie à la théorie des fonctions elliptiques.*"

Maclaurin in his celebrated work *A Treatise on Fluxions*, Edinburgh, 1742, Vol. II, p. 745, shows "how the *elastic curve* may be constructed in all cases by the rectification of the conic sections." On p. 744 he gives Jacob Bernoulli "as the celebrated author who first resolved this as well as several other curious problems" (see *Acta Eruditorium*, 1694, p. 274). It is thus seen that the elliptic integrals made their appearance in the formative period of the integral calculus.

The results that are given in Maclaurin's work were simplified and extended by d'Alembert in his treatise *Recherches sur le calcul intégral. Histoire de l'Ac. de Berlin, Année* 1746, pp. 182–224. The second part of this work, *Des différentielles qui se rapportent à la rectification de l'ellipse ou de l'hyperbole,*