

**AN INTRODUCTION TO
ALGEBRA UPON
THE INDUCTIVE
METHOD OF INSTRUCTION**

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An introduction to algebra upon the inductive method of instruction by Warren Colburn

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WARREN COLBURN

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A L G E B R A
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BY WARREN COLBURN,
AUTHOR OF INTELLECTUAL ARITHMETIC AND SEQUEL TO DITTO.



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DISTRICT OF MASSACHUSETTS, to wit.

District Clerk's Office.

BE IT REMEMBERED, That on the twenty-fourth day of June, A. D. 1825, in the forty-ninth year of the Independence of the United States of America, WARREN COLBURN, of the said district, has deposited in this office the title of a book, the right whereof he claims as author, in the words following, to wit:—

“ An Introduction to Algebra, upon the Inductive Method of Instruction. By Warren Colburn, Author of First Lessons in Arithmetic, &c.”

In conformity to the act of the Congress of the United States, entitled “ An act for the encouragement of learning, by securing the copies of maps, charts, and books, to the authors and proprietors of such copies, during the times therein mentioned;” and also to an act, entitled “ An act supplementary to an act, entitled An act for the encouragement of learning, by securing the copies of maps, charts, and books, to the authors and proprietors of such copies, during the times therein mentioned, and extending the benefits thereof to the arts of designing, engraving, and etching historical and other prints.”

JNO. W. DAVIS,

Clerk of the District of Massachusetts.

PREFACE.



THE first object of the author of the following treatise has been to make the transition from arithmetic to algebra as gradual as possible. The book, therefore, commences with practical questions in simple equations, such as the learner might readily solve without the aid of algebra. This requires the explanation of only the signs plus and minus, the mode of expressing multiplication and division, and the sign of equality ; together with the use of a letter to express the unknown quantity. These may be understood by any one who has a tolerable knowledge of arithmetic. All of them, except the use of the letter, have been explained in arithmetic. To reduce such an equation requires only the application of the ordinary rules of arithmetic ; and these are applied so simply, that scarcely any one can mistake them, if left entirely to himself. One or two questions are solved first with little explanation in order to give the learner an idea of what is wanted, and he is then left to solve several by himself.

The most simple combinations are given first, then those which are more difficult. The learner is expected to derive most of his knowledge by solving the examples himself ; therefore care has been taken to make the explanations as few and as brief as is consistent with giving an idea of what is required.

In fact, explanations rather embarrass than aid the learner, because he is apt to trust too much to them, and neglect to employ his own powers ; and because the explanation is frequently not made in the way, that would naturally suggest itself to him, if he were left to examine the subject by himself. The best mode, therefore, seems to be, to give examples so simple as to require little or no explanation, and let the learner reason for himself, taking care to make them more difficult as he proceeds. This method, besides giving the learner confidence, by making him rely on his own powers, is much more interesting to him, because he seems to himself to be constantly making new discoveries. Indeed, an apt scholar will frequently make original explanations much more simple than would have been given by the author.

This mode has also the advantage of exercising the learner in reasoning, instead of making him a listener, while the author reasons before him.

The examples in the first fifty pages involve nearly all the operations, that are ever required in simple numerical equations, with one and two unknown quantities.

In the ninth article, the learner is taught to generalize particular cases, and to form rules. Here he is first taught to represent known quantities by letters, and at the same time the purpose of it. The transition from particular cases to general principles is made as gradual as possible. At first only a part of the question is generalized, and afterwards the whole of it.

When the learner understands the purpose of representing known quantities as well as unknown, by letters or general symbols, he is considered as fairly introduced to the subject of algebra, and ready to commence where the subject is usually commenced in other treatises. Accordingly he is taught the fundamental rules, as applied to literal quantities. Much of this however is only a recapitulation in a general form, of what he has previously learnt, in a particular form.

After this, various subjects are taken up and discussed. There is nothing peculiar in the arrangement or in the manner of treating them. The author has used his own language, and explained as seemed to him best, without reference to any other work. A large number of examples introduce and illustrate every principle, and as far as seemed practicable, the subjects are taught by example rather than by explanation.

The demonstration of the Binomial Theorem is entirely original, so far as regards the rule for finding the coefficients. The rule itself is the same that has always been used. The manner of treating and demonstrating the principle of summing *series by difference*, is also original.*

Proportions have been discarded in algebra as well as in arithmetic. The author intended to give, in an appendix, some directions for using proportions, to assist those who might have occasion to read other treatises on mathematics. But this volume was already too large to admit it. It is believed, however, that few will find any difficulty in this respect. If they do, one hour's study of some treatise which explains proportions will remove it.

* See Boston Journal of Philosophy and the Arts, No. 5, for May, 1825.

In order to study this work to advantage, the learner should solve every question in course, and do it *algebraically*. If he finds a question which he can solve as easily without the aid of algebra as with it, he may be assured, this is what the author expected. If he first solves a question, which involves no difficulty, he will understand perfectly what he is about, and he will thereby be enabled to encounter those which are difficult.

When the learner is directed to turn back and do in a new way, something he has done before, let him not fail to do it, for it will be necessary to his future progress ; and it will be much better to trace the new principle in what he has done before, than to have a new example for it.

The author has heard it objected to his arithmetics by some, that they are too easy. Perhaps the same objection will be made to this treatise on algebra. But in both cases, if they *are* too easy, it is the fault of the subject, and not of the book. For in the *First Lessons*, there is no explanation ; and in the *Sequel* there is probably less than in any other books, which explain at all. As easy however as they are, the author believes that whoever undertakes to teach them, will find the intellects of his scholars more exercised in studying them, than in studying the most difficult treatise he can put into their hands. When the learner feels, that the subject is above his capacity, he dares not attempt any thing himself, but trusts implicitly to the author ; but when he finds it level with his capacity, he readily engages in it. But here there is something more. The learner is required to perform a part himself. He finds a regular part assigned to him, and if the teacher does his duty, the learner must give a great many explanations which he does not find in the book.

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