ELEMENTS OF THE CONIC SECTIONS

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BOSTON AND NEW YORK.

PREFACE.

For two years I have been giving my advanced classes in Geometry work in conic sections by means of notes copied by the hektograph. The amount of work has grown on my hands till it has become too large for convenient use in its present form; so it is now put in the hands of the printer.

To show the connection of the curves by their definition, and not to follow too closely the line of reasoning in other American works on this subject, I used Boscovitch's definitions, and find by so doing many theorems can be demonstrated with less work than by using the ordinary definitions.

The demonstrations are necessarily longer than those relating to the straight line and circle, but there is a corresponding advantage in discipline to the student.

Although more details are given than in English works, it is hoped that enough has been omitted to require the independent thought of the student; and the author trusts it may be of some use to those desiring a knowledge of this extensive subject.

Kingston, Pa.,
Jan. 28, 1887.

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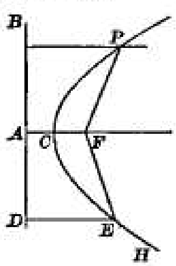
DEFINITIONS.

A curved line is a line no part of which is straight, and points on the curve are so situated that they answer the requirements of some definition. To define the curved lines called conic sections, we assume a fixed straight line, called a *directrix*, and a fixed point without this line, called a *focus*.

A conic section is a curved line the distance from any point in which to the directrix is in a constant ratio to its distance from the focus.

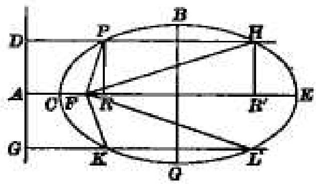
There are three classes of curves coming under this definition.

The parabola, in which any point is the same distance from the directrix as from the focus; *i.e.*, the constant ratio of the above definition equals unity.



Take BD as the directrix, and F as the focus of the parabola PCH; then $BP = PF$ and $DE = EF$.

The ellipse, in which any point is farther from the directrix than from the focus; *i.e.*, the constant ratio is greater than unity.

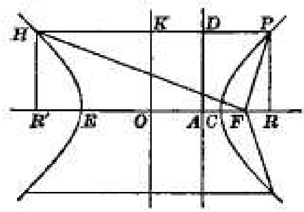


Take DG as the directrix, and F as the focus; then

$$\frac{DP}{PF}=\frac{DH}{HF}=\frac{GK}{KF}=\frac{GL}{LF},$$

and $DP > PF.$

The hyperbola, in which any point is nearer the directrix than the focus; *i.e.*, the constant ratio is less than unity.



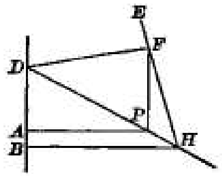
Take DA as the directrix, and F the focus of the hyperbola $PHEC$; then

$$\frac{DP}{PF}=\frac{DH}{HF},$$

and $DP < PF.$

Problem I.

Having given a point on a line, to find a second point on the line such that the ratio of its distances from a fixed line (the directrix) and from a fixed point (the focus) shall equal the ratio of the corresponding distances from the given point.



Let DB be the directrix, and F the focus. Take P as the given point on the line DH, and suppose the problem solved, and H to be the required point; then

$$AP : PF :: BH : HF; \qquad (1)$$

or, by alternation,

$$AP : BH :: PF : HF; \qquad (2)$$

AP and BH being perpendicular to DB by construction,

$$AP : BH :: DP : DH; \qquad (3)$$

comparing (2) and (3),

$$PF : HF :: DP : DH;$$

therefore DF bisects the angle PFE.

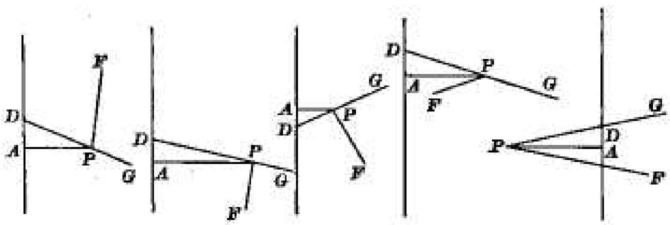
Hence, to find the required point, join the focus F with the point D in which the line intersects the directrix, and draw a line making an angle DFE equal to DFP: the point in which this line EF meets the given line is the required point.

Corollary I. As but one line can be drawn from F, making with DF an angle equal to DFP, there are but two points on a line from which a given ratio, formed as in the problem, can be made.

Corollary II. If the point P be moved along the line DH so that DFP approaches a right angle, its equal DFE

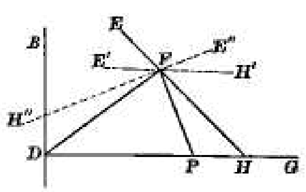
also approaches a right angle, and when each becomes a right angle, the line EFH coincides with FP, and H with P; hence there will be but one point on a line from which a given ratio can be formed when the line joining this point to the focus is perpendicular to the line joining the focus to the intersection of the given line and the directrix.

Exercises. Find a second point on the line DG of each of the following figures, so that the ratio of the lines corresponding to AP and PF shall equal their ratio.



Theorem I.

A line perpendicular to the directrix will meet a parabola in one point, it may meet an ellipse in two points, and it will meet a hyperbola in two points.



Take BD as the directrix, F the focus, and P a point of a conic section on the line DG perpendicular to BD. Make $DFE = DFP$, to find the second point of the conic on DG (Prob. I.).