A TREATISE ON THE MOTION OF VORTEX RINGS: AN ESSAY TO WHICH THE ADAMS PRIZE WAS ADJUDGED IN 1882, IN THE UNIVERSITY OF CAMBRIDGE

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J. J. THOMSON

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BY

J. J. THOMSON, M.A.

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PREFACE.

THE subject selected by the Examiners for the Adams Prize for 1882 was

"A general investigation of the action upon each other of two closed vortices in a perfect incompressible fluid."

In this essay, in addition to the set subject, I have discussed some points which are intimately connected with it, and I have endeavoured to apply some of the results to the vortex atom theory of matter.

I have made some alterations in the notation and arrangement since the essay was sent in to the Examiners, in so doing I have received great assistance from Prof. G. H. Darwin, F.R.S. one of the Examiners, who very kindly lent me the notes he had made on the essay. Beyond these I have not made any alterations in the first three parts of the essay: but to the fourth part, which treats of a vortex atom theory of chemical action, I have made some additions in the hope of making the theory more complete: paragraph 60 and parts of paragraphs 58 and 59 have been added since the essay was sent in to the Examiners.

I am very much indebted to Prof. Larmor of Queen's College, Galway, for a careful revision of the proofs and for many valuable suggestions.

J. J. THOMSON.

TRINITY COLLEGE, CAMBRIDGE.
October 1st, 1883.

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$$\frac{1}{(1+\alpha^2-2\alpha\cos\theta)^{\frac{1}{2}}}$$

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