A TEXT-BOOK OF GEOMETRY

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G. A. WENTWORTH

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GEOMETRY.

REVISED EDITION.

BY

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PREFACE.

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MOST persons do not possess, and do not easily acquire, the power of abstraction requisite for apprehending geometrical conceptions, and for keeping in mind the successive steps of a continuous argument. Hence, with a very large proportion of beginners in Geometry, it depends mainly upon the *form* in which the subject is presented whether they pursue the study with indifference, not to say aversion, or with increasing interest and pleasure.

In compiling the present treatise, the author has kept this fact constantly in view. All unnecessary discussions and scholia have been avoided; and such methods have been adopted as experience and attentive observation, combined with repeated trials, have shown to be most readily comprehended. No attempt has been made to render more intelligible the simple notions of position, magnitude, and direction, which every child derives from observation; but it is believed that these notions have been limited and defined with mathematical precision.

A few symbols, which stand for words and not for operations, have been used, but these are of so great utility in giving *style* and *perspicuity* to the demonstrations that no apology seems necessary for their introduction.

Great pains have been taken to make the page attractive. The figures are large and distinct, and are placed in the middle of the page, so that they fall directly under the eye in immediate connection with the corresponding text. The *given* lines of the figures are full lines, the lines employed as *aids* in the demonstrations are short-dotted, and the *resulting* lines are long-dotted.

In each proposition a concise statement of what is given is printed
in one kind of type, of what is required in another, and the demon-
stration in still another. The reason for each step is indicated in
small type between that step and the one following, thus preventing
the necessity of interrupting the process of the argument by referring
to a previous section. The number of the section, however, on which
the reason depends is placed at the side of the page. The constituent
parts of the propositions are carefully marked. *Moreover, each distinct
assertion in the demonstrations and each particular direction in the
construction of the figures, begins a new line; and in no case is it
necessary to turn the page in reading a demonstration.*

This arrangement presents obvious advantages. The pupil perceives
at once what is given and what is required, readily refers to the figure
at every step, becomes perfectly familiar with the language of Geom-
etry, acquires facility in simple and accurate expression, rapidly *learns
to reason*, and lays a foundation for completely establishing the
science.

Original exercises have been given, not so difficult as to discourage
the beginner, but well adapted to afford an effectual test of the degree
in which he is *mastering* the subjects of his reading. Some of these
exercises have been placed in the early part of the work in order
that the student may discover, at the outset, that to commit to mem-
ory a number of theorems and to reproduce them in an examination
is a useless and pernicious labor; but to learn their uses and appli-
cations, and to acquire a readiness in exemplifying their utility is to
derive the full benefit of that mathematical training which looks not
so much to the *attainment of information* as to the *discipline of the
mental faculties.*

<div style="text-align: right">G. A. WENTWORTH.</div>

PHILLIPS EXETER ACADEMY,
    1878.

TO THE TEACHER.

WHEN the pupil is reading each Book for the first time, it will be well to let him write his proofs on the blackboard in his own language; care being taken that his language be the simplest possible, that the arrangement of work be vertical (without side work), and that the figures be accurately constructed.

This method will furnish a valuable exercise as a language lesson, will cultivate the habit of neat and orderly arrangement of work, and will allow a brief interval for deliberating on each step.

After a Book has been read in this way, the pupil should review the Book, and should be required to draw the figures free-hand. He should state and prove the propositions orally, using a pointer to indicate on the figure every line and angle named. He should be encouraged, in reviewing each Book, to do the original exercises; to state the converse of propositions; to determine from the statement, if possible, whether the converse be true or false, and if the converse be true to demonstrate it; and also to give well-considered answers to questions which may be asked him on many propositions.

The Teacher is strongly advised to illustrate, geometrically and arithmetically, the principles of limits. Thus a rectangle with a constant base b, and a variable altitude x, will afford an obvious illustration of the axiomatic truth that the product of a constant and a variable is also a variable; and that the limit of the product of a constant and a variable is the product of the constant by the limit of the variable. If x increases and approaches the altitude a as a limit, the area of the rectangle increases and approaches the area of the rectangle ab as a limit; if, however, x decreases and approaches zero as a limit, the area of the rectangle decreases and approaches zero for a limit. An arithmetical illustration of this truth may be given by multiplying a constant into the approximate values of any repetend. If, for example, we take the constant 60 and the repetend 0.3333, etc., the approximate values of the repetend will be $\frac{3}{10}$, $\frac{33}{100}$,

$\frac{111}{1000}$, $\frac{1111}{10000}$, etc., and these values multiplied by 60 give the series 18, 19.8, 19.98, 19.9998, etc., which evidently approaches 20 as a limit; but the product of 60 into $\frac{1}{3}$ (the limit of the repetend 0.333, etc.) is also 20.

Again, if we multiply 60 into the different values of the decreasing series $\frac{1}{30}$, $\frac{1}{300}$, $\frac{1}{3000}$, $\frac{1}{30000}$, etc., which approaches zero as a limit, we shall get the decreasing series 2, $\frac{1}{5}$, $\frac{1}{50}$, $\frac{1}{500}$, etc.; and this series evidently approaches zero as a limit.

In this way the pupil may easily be led to a complete comprehension of the subject of limits.

The Teacher is likewise advised to give frequent written examinations. These should not be too difficult, and sufficient time should be allowed for accurately constructing the figures, for choosing the best language, and for determining the best arrangement.

The time necessary for the reading of examination-books will be diminished by more than one-half, if the use of the symbols employed in this book be allowed.

G. A. W.

PHILLIPS EXETER ACADEMY,
1879.

NOTE TO REVISED EDITION.

THE first edition of this Geometry was issued about nine years ago. The book was received with such general favor that it has been necessary to print very large editions every year since, so that the plates are practically worn out. Taking advantage of the necessity for new plates, the author has re-written the whole work; but has retained all the distinguishing characteristics of the former edition. A few changes in the order of the subject-matter have been made, some of the demonstrations have been given in a more concise and simple form than before, and the treatment of Limits and of Loci has been made as easy of comprehension as possible.

More than seven hundred exercises have been introduced into this edition. These exercises consist of theorems, loci, problems of construction, and problems of computation, *carefully graded and specially adapted to beginners*. No geometry can now receive favor unless it provides exercises for independent investigation, which must be of such a kind as to interest the student as soon as he becomes acquainted with the methods and the spirit of geometrical reasoning. The author has observed with the greatest satisfaction the rapid growth of the demand for original exercises, and he invites particular attention to the systematic and progressive series of exercises in this edition.

The part on Solid Geometry has been treated with much greater freedom than before, and the formal statement of the reasons for the separate steps has been in general omitted, for the purpose of giving a more elegant form to the demonstrations.

A brief treatise on Conic Sections (Book IX) has been prepared, and is issued in pamphlet form, at a very low price. It will also be bound with the Geometry if that arrangement is found to be generally desired.

The author takes this opportunity to express his grateful appreciation of the generous reception given to the Geometry heretofore by the great body of teachers throughout the country, and he confidently anticipates the same generous judgment of his efforts to bring the work up to the standard required by the great advance of late in the science and method of teaching.

The author is indebted to many correspondents for valuable suggestions; and a special acknowledgment is due, for criticisms and careful reading of proofs, to Messrs. C. H. Judson, of Greenville, S.C.; Samuel Hart, of Hartford, Conn.; J. M. Taylor, of Hamilton, N.Y.; W. Le Conte Stevens, of Brooklyn, N.Y.; E. R. Offutt, of St. Louis, Mo.; J. L. Patterson, of Lawrenceville, N.J.; G. A. Hill, of Cambridge, Mass.; T. M. Blakslee, Des Moines, Ia.; and G. W. Sawin, of Cambridge, Mass.

Corrections or suggestions will be thankfully received.

G. A. WENTWORTH.

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    1888.