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W. J. MILLER

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MATHEMATICAL QUESTIONS,

WITH THEIR

SOLUTIONS,

FROM THE "EDUCATIONAL TIMES,"

WITH MANY

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EDITED BY

W. J. MILLER, B.A.,

MATHEMATICAL MASTER, HUDDERSFIELD COLLEGE.

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LIST OF CONTRIBUTORS.

ALEXANDER, F., Gorton, Manchester.

ANDERSON, D. M., Kirriemuir, Scotland.

BALL, Professor ROBERT S., M.A., Dublin.

BATTAGLINI, GIUSEPPE, Professor of Mathematics in the University of Naples.

BERRIMAN, J. S., Gloucester.

BILLS, SAMUEL, Newark-on-Trent.

BLAKEMORE, J. W. T., B.A., Stafford.

BLISSARD, Rev. J., B.A., The Vicarage, Hampstead Norris, Berks.

BOOTH, Rev. Dr., F.R.S., The Vicarage, Stone, Bucks.

BOURNE, A. A., Atherston.

BOWDITCH, W. L., Wakefield.

BRANQUART, PAUL, Church House, Ealing.

BROMFIELD, S. W., Christ Church College, Oxford.

BROWN, A. CRUM, D.Sc., Edinburgh.

BURNSIDE, W. S., M.A., Trinity College, Dublin.

CASEY, JOHN, B.A., Kingstown, Ireland.

CAYLEY, A., F.R.S., Sadlerian Professor of Mathematics in the University of Cambridge; Corresponding Member of the Institute of France.

CHADWICK, W., Oxford.

CLARKE, Captain A. R., R.E., F.R.S., Ordnance Survey Office, Southampton.

CLIFFORD, W. K., M.A., Fellow of Trinity College, Cambridge.

COCKLE, Sir J., M.A., F.R.S., Chief Justice of Queensland; President of the Queensland Philosophical Society.

COHEN, ARTHUR, M.A., London.

COLLINS, MATTHEW, B.A., Dublin.

CONWILL, J., Leighlinbridge, Ireland.

CONOLLY, E., Mountnugent, Ireland.

COTTERILL, THOS., M.A., London, late Fellow of St. John's College, Cambridge.

CREMONA, LUIGI, Professor, R. Istituto Tecnico Superiore di Milano.

CROFTON, M. W., B.A., F.R.S., Prof. of Math. in the Royal Military Acad., Woolwich.

DALE, JAMES, Aberdeen.

DAVIS, WILLIAM BARRETT, B.A., London.

DE MORGAN, AUGUSTUS, F.R.A.S., London.

DOBSON, T., B.A., Head Master of Hexham Grammar School.

DUPAIN, J. C., Professeur au Lycée d'Angoulême.

EASTERBY, W., B.A., Grammar School, St. Asaph.

EVANS, ASHER B., M.A., Lockport, New York, United States.

EVERETT, J. D., D.C.L., Professor of Nat. Phil. in the Queen's University, Belfast.

FENWICK, STEPHEN, F.R.A.S., Mathematical Master in the R. M. Acad., Woolwich.

FERRERS, Rev. N. M., M.A., Caius College, Cambridge.

FICKLIN, Professor, Philadelphia, United States.

FITZGERALD, E., Bagenalstown, Ireland.

FLOOD, P. W., Ballingarry, Ireland.

GARDINER, MARTIN, late Professor of Mathematics in St. John's College, Sydney.

GENESE, R. W., St. John's College, Cambridge.

GERAGHTY, W., Dublin.

GODFRAY, HUGH, M.A., Cambridge.

GODWARD, WILLIAM, Law Life Office, London.

GREENWOOD, JAMES M., Kirksville, Missouri, United States.

GREER, H. R., B.A., Mathematical Master in the R. M. (Cadets') Coll., Sandhurst.

GRIFFITH, W., Fairfield, Ohio, United States.

GRIFFITHS, J., M.A., Fellow of Jesus College, Oxford.

HALL, H. S., M.A., Clifton College.

HANLON, G. O., Dublin.

HANNA, W., Literary Institute, Belfast.

HARLEY, Rev. ROBERT, F.R.S., Leicester.

HART, Dr. D. S., Stonington, Connecticut, United States.

HERMITE, Ch., Membre de l'Institut, Paris.

HILL, Rev. E., M.A., St. John's College, Cambridge.

HIRST, Dr. T. A., F.R.S., Registrar of the University of London.

HOPPS, WILLIAM, Leonard Street, Hull.

HOPKINS, Rev. G. H., M.A., Claughton, Birkenhead.

HOSKINS, HENRY, Islington, London.

HUDSON, C. T., LL.D., Manilla Hall, Clifton.

HUDSON, W. H. H., M.A., Fellow of St. John's College, Cambridge.

INGLEBY, C. M., M.A., LL.D., London.

JENKINS, MORGAN, M.A., late Scholar of Christ's College, Cambridge.

KIRKMAN, Rev. T. P., M.A., F.R.S., Croft Rectory, near Warrington.

KITCHENER, F. E., M.A., Rugby School.

KITCHIN, Rev. J. L., M.A., Head Master of Bideford Grammar School.

KNISELY, Rev. U. J., Newcomerstown, Ohio, United States.

LAVERTY, W. H., B.A., Queen's College, Oxford.

LAW, C., Cambridge.

LEVY, W. H., Shalbourne, Berks.

MADDEN, W. M., Trinity Parsonage, Wakefield.

MANNHEIM, M., Professeur à l'Ecole Polytechnique, Paris.

MARTIN, ARTEMAS, Mathematical Editor of "The Visitor," McKean, Erie Co., Pennsylvania, United States.

MARTIN, Rev. H., M.A., Examiner in Mathematics and Natural Philosophy in the University of Edinburgh.

MASON, J., East Castle Colliery, near Newcastle-on-Tyne.
MATHEWS, F. C., M.A., London.
MATTESON, Dr. JAMES, De Kalb Centre, Illinois, United States.
McCAY, W. S., B.A., Trinity College, Dublin.
McCOLL, HUGH, Boulogne-sur-Mer, France.
McCORMICK, E., Ledbury, Hereford.
McDOWELL, J., M.A., F.R.A.S., Pembroke College, Cambridge.
McNEILL, JAMES A., Belfast.
MERRIFIELD, C. W., F.R.S., Principal of the Royal School of Naval Architecture, South Kensington.
MERRIFIELD, J., Ph.D., F.R.A.S., Science School, Plymouth.
MILLER, W. J., B.A., 11, Westfield Terrace, Huddersfield.
MINCHIN, G. M., B.A., Trinity College, Dublin.
MITCHESON, T., The Cedars, Rickmansworth.
MOON, ROBERT, M.A., London, late Fellow of Queen's College, Cambridge.
MOULTON, J. F., M.A., Christ's College, Cambridge.
MURPHY, HUGH, Pembroke Road, Dublin.
NELSON, R. J., M.A., Sailor's Institute, Naval School, London.
O'CAVANAGH, PATRICK, Dublin.
OGILVIE, G. A., Leiston, near Saxmundham.
OTTER, W. CURTIS, F.R.A.S., Liverpool.
PANTON, A. W., B.A., Trinity College, Dublin.
POLIGNAC, Prince Camille de, Paris.
RENSHAW, S. A., Elm Avenue, New Basford, Nottingham.
RIPPIN, Charles, R.,M.A., Woolwich Common.
ROBERTS, SAMUEL, M.A., London.
ROBERTS, Rev. W., M.A., Fellow and Senior Tutor, Trinity College, Dublin.
ROBERTS, W., junior, Trinity College, Dublin.
ROCKER, A. W., B.A., Brasenose College, Oxford.
RUTHERFORD, Dr., F.R.A.S., Woolwich.
SALMON, Rev. G., D.D., F.R.S., Fellow of Trinity College, Dublin.
SANDERS, J. B., Cloverport, Kentucky, United States.
SANDERSON, Rev. T. J., M.A., Litlington Vicarage, Royston.
SAVAGE, THOMAS, M.A., Fellow of Pembroke College, Cambridge; Mathematical Master in the Royal Military (Staff) College, Sandhurst.
SCOTT, J., Judge of the Ohio Supreme Court, Bucyrus, United States.
SHARPE, Rev. H. T., M.A., Vicar of Cherry Marham, Norfolk.
SIVERLY, WALTER, Oil City, Pennsylvania, United States.
SPOTTISWOODE, WILLIAM, M.A., F.R.S., Grosvenor Place, London.
SPRAGUE, THOMAS BOND, M.A., London.
STANLEY, ARCHER, London.
SWAINSON, T., Cleator, near Whitehaven.
SYMES, R. W., B.A., London.
SYLVESTER, J. J., LL.D., F.R.S., Corresponding Member of the Institute of France.
TAIT, P. G., M.A., Professor of Natural Philosophy in the University of Edinburgh.
TARLETON, FRANCIS A., M.A., Fellow of Trinity College, Dublin.
TAYLOR, C., M.A., Fellow of St. John's College, Cambridge.
TAYLOR, H. M., B.A., Fellow of Trinity College, Cambridge; Vice-Principal of the Royal School of Naval Architecture, South Kensington.
TAYLOR, J. H., B.A., Cambridge.
TEBAY, SEPTIMUS, B.A., Head Master of Rivington Grammar School.
THOMSON, F. D., M.A., Exeter.
TODHUNTER, ISAAC, F.R.S., St. John's College, Cambridge.
TOMLINSON, H., Christ Church College, Oxford.
TORELLI, GABRIEL, Naples.
TORRY, Rev. A. F., M.A., St. John's College, Cambridge.
TOWNSEND, Rev. R., M.A., F.R.S., Fellow of Trinity College, Dublin.
TUCKER, R., M.A., Mathematical Master in University College School, London.
TURRELL, I. H., Cumminsville, Ohio, United States.
VOSE, Prof. G. B., Washington and Jefferson College, Washington, United States.
WALKER, J. J., M.A., Vice-Principal of University Hall, London.
WALMSLEY, J., Manchester.
WARREN, R., M.A., Trinity College, Dublin.
WATSON, STEPHEN, Haydonbridge, Northumberland.
WHITE, Rev. J., M.A., Brook Hill Park, Plumstead.
WHITWORTH, Rev. W. A., M.A., Fellow of St. John's College, Cambridge.
WILKINSON, Rev. M. M. U., Reepham Rectory, Norwich.
WILKINSON, T. T., F.R.A.S., Burnley.
WILLIAMSON, B., M.A., Fellow of Trinity College, Dublin.
WILSON, JAMES, Cork.
WILSON, Rev. R., D.D., Chelsea.
WOLSTENHOLME, Rev. J., M.A., Fellow of Christ's College, Cambridge.
WOOLHOUSE, W. S. B., F.R.A.S., &c., Alwyne Lodge, Canonbury, London.
YOUNG, J. R., Priory Cottage, Peckham.

Contributors deceased since the Publication of Vol. I.

DE MORGAN, G. C., M.A.; HOLDITCH, Rev. H., M.A.; LEA, W.; O'CALLAGHAN, J.; PURKISS, H. J., B.A.; PROUHET, E.; SADLER, G. T., F.R.A.S.; WRIGHT, Rev. R. H., M.A.

CONTENTS.

Mathematical Papers, &c.

No.		Page
68.	An easy way of remembering the Formulæ of Reduction in the Integral Calculus, both as regards their Investigation and their Application. By H. McColl.	43
69.	Magic Squares for 1870. By Artemas Martin.	46
70.	To prove that the middle points of the three Diagonals of a complete Quadrilateral are in one straight line. By J. A. McNeill.	47
71.	On the Exponential Theorem. By Artemas Martin.	48
72.	Note on Question 3167. By the Rev. T. P. Kirkman, M.A., F.R.S.	49
73.	Note on Question 2986. By C. W. Merrifield, F.R.S.	63
74.	Note on Question 3232. By G. O. Hanlon.	69
75.	Note on the Diffusion of Gases. By G. O. Hanlon.	98

Solved Questions.

No. Page

2023. If n be a prime number,

$$\lfloor n \left\{ \frac{1}{n} - \frac{1}{n-1} + \frac{1}{\lfloor 2\,(n-2)} - \frac{1}{\lfloor 3\,(n-3)} + \dots + \frac{1}{\lfloor n-1} - \frac{1}{\lfloor n} \right\}$$

is a positive integer of the form M $(n)-2$. 112

2111. In a Cartesian oval whose two internal foci coincide, the difference of the two arcs intercepted by any two transversals from the external focus is equal to a straight line which may be found. 21

2151. In a Cartesian oval with a finite node (a nodal Limaçon), the difference of the lengths of the loops is four times the distance between the vertices. 101

2279. An ellipse is described having its major axis three times the minor. Any parabola is drawn satisfying the following conditions:—The intercept made on its principal axis by the axes of the ellipse is constant, and equal to the sum of the semi-axes of the ellipse; its focus lies on the ellipse, in the same

No. Page

quadrant as the intercept, and the tangent at its vertex passes through the foot of the perpendicular from the middle of the intercept on the major axis of the ellipse. An equilateral hyperbola is also drawn, whose axis is equal to the intercept, and whose axis and centre fall on the major axis and centre of the ellipse. Now if any tangent is drawn to the parabola, and from the points in which it meets the hyperbola lines are drawn parallel to the asymptotes, one of the loci of their intersections envelopes a circle. 108

2360. Find the average area of all the triangles in which there are given—(1) an angle and the sum of the two containing sides, all possible values of these sides being considered equally probable; (2) the sum of two sides only, all possible values of these two sides, and of the included angle, being considered equally probable; (3) one side and the sum of the other two; (4) the sum of the three sides; all possible values of the sides in (3) and (4) being considered equally probable. Also show (5) that the average of the averages in (3) is seven-eighths of the average in (4). 103

2547. Find the average area of the quadrilaterals whose angular points are the centre of a given circle and three points taken at random on its circumference. 29

2550. Five circles, S', S'', S_1, S_2, R, are drawn, S' and S'' being concentric, S_1 and S_2 likewise concentric, and the difference between the radii of S' and S'' being equal to the difference between those of S_1 and S_2. Then, if two ellipses are drawn, one having for foci the centres of S' and R, and passing through the intersections of these circles, the other similarly related to S'' and S_1, the ellipses will intersect in two points on a hyperbola having for foci the centres of the circles R and S_2, and passing through their intersections. 53

2596. Two particles are projected with the same velocity, in the same direction, at right angles to the line joining them: find the curves they will describe if they attract according to the law of nature. If the particles were equal, find what law of resistance, in the medium through which they pass, would cause them to describe parts of the same circle. 88

2652. If $l+m+n=0$, prove that

$$(l^2+m^2+n^2)^3-2\,(l-m)^2\,(m-n)^2\,(n-l)^2-54\,l^2m^2n^2$$

vanishes; and hence verify the identity $\Delta = I^3-27J^2$, where Δ is the discriminant, and I, J are the fundamental invariants of $(a, b, c, d, e \between x, y)^4$. 93

2790. Find the mean distance of all the points in a right circular cylinder from one end of the axis. 64

2791. 1. In the sides BC, CA, AB of the triangle ABC points D, E, F are respectively taken at random, and DEF is an inscribed triangle. Again, in the triangles AEF, BFD, CDE, points P, Q, R are respectively taken at random. Show that the average area of the triangle PQR is one-fourth that of ABC.

2. If G_1, G_2, G_3 be the centres of gravity of the triangles AEF, BFD, CDE; then generally $9\Delta G_1G_2G_3 = 2\Delta ABC + \Delta DEF$. ... 19

No. Page

2834. Show that the equation to the envelope referred to in Question 2780 (see *Reprint*, Vol. XI., p. 63), may be thrown into the form $U^2 + V^2 = 0$, where $U = 0$ is the equation to a similar and coaxal ellipse, $V = 0$ that to its equi-conjugate diameters. 91

2852. 1. The locus of the centre of circles passing through four points, related as A, B, C, D in Questions 2803, 2833 (*Reprint*, Vol. XII., pp. 37, 90), is the conic polar reciprocal to the given conic with respect to the concentric circle bisecting the intervals between the centre and foci.

2. The normal at B, and the line joining the centre of the circle which osculates the conic at that point with the centre of the circle ABCD, make equal angles with the axis of the conic. 96

2865. Determine at what points on an ellipsoid the distance of the normal from the centre is a maximum. 69

2936. A heavy particle is projected so as to move on a rough inclined plane whose angle of inclination is equal to the angle of friction; prove that the hodograph of its path is a parabola, and the intrinsic equation of the path is

$$\frac{ds}{d\phi} = \frac{u^2}{4g \sin \alpha} \left\{ \frac{3 \sin \phi + 2 \sin^2 \phi}{(1 + \sin \phi)^2} + \log \tan \left(\tfrac{1}{4}\pi + \tfrac{1}{2}\phi\right) \right\},$$

u being the velocity at the highest point, and $\tan \alpha$ the coefficient of friction. 31

2944. If O be that point in the normal to a parabola at P through which, if any chord pass, it will subtend a right angle at P, PO will be bisected by the axis. 57

2953. Eliminate y from the equations $x^x + y^y = a$, $x^y + y^x = b$. 48

2956. Find the locus of a point such that, if from it tangents be drawn to a central conic, the product of these tangents shall be in a constant ratio to the product of the focal distances....... 100

2957. ABC and A′B′C′ are two co-polar triangles. Let the straight line formed by joining the points of intersection of CB and C′A′, and of C′B′ and CA be denoted by DE. Let O be the intersection of BC and B′C′, then the straight line joining O to the intersection of CC′ and DE is the fourth harmonic to BC, B′C′ and the axis of the triangles. .... 34 & 101

2968. In Question 2947, show (1) that the point of intersection of the normals at B, C, D lies on the normal at the extremity of the diameter through A; and (2) that the radical axes of the osculating circles at B, C, D are tangents to the curve

$$\left(\frac{x}{2a}\right)^{\frac{2}{3}} + \left(\frac{y}{2b}\right)^{\frac{2}{3}} = 1 \text{..........................} \quad 87$$

2980. A given curve moves without rotation so as always to touch a fixed curve; in any position draw the common tangent, also a second tangent to the moving curve at right angles to the former: find the envelope of the second tangent. Show that, if the intrinsic equation of the fixed curve be $\rho = f(\theta)$, and that of the moving curve $\rho = \phi(\theta)$, that of the required envelope

will be $$\rho = \frac{d}{d\theta} f\left(\theta - \tfrac{1}{2}\pi\right) - \frac{d}{d\theta} \phi\left(\theta - \tfrac{1}{2}\pi\right) + \phi(\theta);$$

No.		Page
	where ρ in all cases stands for the radius of curvature, and θ for the inclination of the tangent to a fixed axis.	98
2981.	If the sum of the squares of two consecutive integers be equal to the square of another integer, find their general values, and show how to find any number of particular solutions.	89
2986.	Two diameters of an oval bisect one another at right angles. Supposing the diameters given, required the form of the oval for which the ratio of the area to the perimeter is the greatest possible.	62
2988.	A, B, C, D throw with three dice in succession for a prize, the highest throw winning, and equal throws continuing the trial. At the first throw A throws 13. Find his chance of winning.	86
2991.	Through the extremities of the major axis of an ellipse two lines are drawn in random directions: find the chance of their intersecting within the ellipse.	33
3002.	If every two of five circles A, B, C, D, E touch each other, except D and E, prove that the common tangent of D and E is just twice as long as it would be if D and E touched each other.	17
3004.	A particle describes an ellipse about a centre of force in the focus S. Show that its mean distance from S is $a(1+\frac{1}{2}e^2)$. ...	100
3005.	Find the mean distance of all the points in a parallelogram from one of its angles.	94
3007.	Ellipses are described having a common transverse axis, and the osculating circles at the extremities of the latera recta are drawn: show that the centres of these circles lie on a septic passing through the centre and extremity of the axis.	63
3030.	Solve the equation $x^3+px^2+qx+r=0$, when (1) ... $q^2-4pqr+8r^2=0$, *Ex.*: $x^3+\frac{153}{80}x^2+5x+4=0$; and (2)...... $p^3-4pq+8r=0$, *Ex.*: $x^3+3x^2+2x-\frac{3}{8}=0$.	65
3031.	Show that the modulus derived from K by a transformation of the order $2n$ (n being any odd integer) is given by the equation $$\mu=\left\{\frac{(1-\Delta am\,\omega)(1-\Delta am\,3\omega)\ldots\{1-\Delta am\,(n-2)\,\omega\}}{(1+\Delta am\,\omega)(1+\Delta am\,3\omega)\ldots\{1+\Delta am\,(n-2)\,\omega\}}\right\}^2,$$ μ being the derived modulus, and $\omega=\frac{K}{n}$, K being, as usual, the complete elliptic function of the first kind.	28
3032.	All circles which touch two given circles are cut orthogonally by the pair of circles which pass through the intersections of the given ones and bisect their angles.	54
3033.	If from the three angles of a triangle circumscribed about a conic straight lines be drawn parallel to those joining the middle points of the opposite sides to the centre of the conic; they will co-intersect.	42
3040.	Required a solution of the biquadratic equation $4x^4+4ax^3+4bx+ab=0$.	31