DESCRIPTION OF A CHRONOGRAPH, ADAPTED FOR MEASURING THE VARYING VELOCITY OF A BODY IN MOTION TROUGH THE AIR AND FOR OTHER PURPOSES

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THE VARYING VELOCITY OF A BODY IN MOTION THROUGH THE AIR,

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FOR OTHER PURPOSES.

BY

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DESCRIPTION OF A CHRONOGRAPH.

THE resistance of fluids to the motion of solids is a problem of great practical and theoretical interest, because sometimes this resistance becomes a source of power, whilst at other times it is a large consumer of power which would otherwise be usefully applied. Thus the action of the wind upon sails is made to drive a mill or a ship. The wind acting upon the sails tends to drive the ship, and the resistance of the water is opposed to the progress of the ship. The resistance of the water to the motion of the oar, the paddle, or the screw, enables the rowers or the steam engine to drive the vessel. A very large part of the power developed in the locomotive is employed in overcoming the resistance of the air to the motion of the train. And quite recently it has been found that the friction of the tidal wave is probably slowly diminishing the velocity of the earth's rotation about its axis. We are dependent upon the resistance of fluids for our power to cross the ocean, for without that property we should not be able to use sail or oar, paddle or screw. Still little is known with accuracy respecting the laws of the resistance of fluids. It is extremely difficult to make satisfactory experiments on account of the great disturbance produced in the surrounding fluid, and as the mathematician knows neither the nature of this disturbance, nor the amount of resistance to be accounted for in particular cases, he is not able to find ground on which to base a satisfactory theory. For a history of what has been done, it will suffice to refer to the articles, "Ballistik" and "Widerstand," in Gehler's Physikalisches Wörterbuch, 1825 and 1842.

The resistance of the air to spherical balls, moving with high velocities, has been a subject of special interest for more than a century, because it was of practical importance to the science of gunnery, and because it offered a simple yet striking instance of the great resistance which a very rare medium would offer to a solid moving in it with a high velocity. Thus Robins[1] states that Dr Halley thought it reasonable to believe that the opposition of the air to large metal shot is scarcely discernible, although in small and light shot, he acknowledged that it ought and must be accounted for. Robins further states,[2] that a musket-ball, fired with a charge of half its own weight of powder, would leave the gun with a velocity of 1700 f.s., and that, with an elevation of 45°, its range would be 17 miles in a vacuum; whilst practical writers on the subject say that the range in air is

[1] Gunnery. Preface, p. 48. [2] p. 145.

short of half a mile. This resistance of the air ρ, depends upon the velocity v, upon the form, and upon the size of the moving body. When a body is at rest in air, the horizontal pressure tending to move it one way is just equal to the pressure tending to move it in the opposite direction. If the body be put in motion by any force, the pressure of the air tending to prevent motion is greater than it was before in that direction, and the pressure in the direction of the motion is less than it was before. The difference of the pressures of the air before and behind the body is called the resistance of the air to that particular form of solid moving with the given velocity: but as the pressure of the air in direction of motion decreases rapidly as the velocity increases, it is commonly neglected.

Thus, ρ is some function of the velocity v, as $f(v)$, such that when the body is at rest, or $v = 0$, we have $\rho = f(v) = 0$. It is usual to assume that ρ can be expanded in a series of the form

$$\rho = av + bv^2 + cv^3 + dv^4 + ev^5 + \&c.$$

where a, b, c, &c. are to be determined by experiment, and are dependent on the form and size of the moving body.

Newton's experiments gave $\rho = bv^2$, and $\therefore\ a = c = d = \&c. = 0$,

Hutton's gave............... $\rho = av + bv^2$, and $\therefore\ c = d = \&c. = 0$,

Didion's gave .... $\rho = bv^2 + cv^3$, and $\therefore\ a = d = \&c. = 0$,

Colonel Mayevski's gave... $\rho = bv^2 + dv^4$, and $\therefore\ a = c = \&c. = 0$.

It is possible that the resistance of the air, for a limited range of variation of velocity, may be tolerably represented by one or more of the above formulæ, but we must be careful not to assume that these formulæ must apply to all velocities of the same body, or that the same law will hold for all forms of bodies. Careful experiments must be made so as to discover some empirical law connecting the resistances of the air to bodies of simple forms and the velocities with which they move. When this is well done, all that is needed for practical use will have been obtained. But, beyond this, the mathematician will have been provided with facts which may serve as tests of the soundness of his theoretical investigations, and in the end he may succeed in forming a theory of resistances on mathematical principles, and thus replace our empirical formulæ with one that is true and general.

Before the discovery of electro-magnetism, the ballistic pendulum of Robins and Hutton afforded the only known practical means for determining the velocities of projectiles and the resistance to their motion exerted by the air. This instrument is so well known that there is no necessity for entering into a minute description of all its parts, and of the precautions to be taken in the use of it. The shot is fired into a body at rest much heavier than itself, to which it communicates motion, so that the two move on together with a common velocity which is capable of being measured. Towards the end of the last century numerous experiments were made with the ballistic pendulum, by Hutton,[1] at Woolwich, with 1, 3, and 6 lbs. spherical balls. Their velocities were measured at distances varying from 30 to 430 feet from the gun. Experiments were also made at Metz with the ballistic pendulum in 1839, &c. with balls of about 8, 12, and 24 lbs.[2] It was found that at 15m. (49 ft.) from the muzzle, the blast from the gun was very sensible. The velocities of the shot were determined at 15, 40, 65, and 90 metres (49, 148, 197, 295 ft.) from the gun. Beyond 295 feet the gun was not sufficiently accurate. Thus the law of the resistance of the air, deduced from the later and more careful experiments, depends upon the loss of velocity of spherical balls moving, at most, through 246 feet. But it is possible to make only one observation on each round. Therefore if it be desired to find the velocity lost by a ball in moving from 15 to 90 metres from the gun, one shot must be fired at the pendulum at a distance of 15 metres, and another at a distance of 90 metres; and, besides the errors in the measurement of the velocities at the moment of impact, there will be a doubt whether the velocity of the second ball, at 15 metres from the gun, was the same as that of the first shot at the same point. The initial velocity therefore enters into the question, and the only thing that can be done in such a case is, to make *numerous* experiments and trust to the *mean* of the results.

Hutton concluded from his experiments that the resistance of the air to a spherical ball $2R$ inches in diameter, was expressed by the formula,[3]

$$\rho = (\cdot0000073\, v - \cdot0015)\, v\, (2R)^2.$$

M. Piobert[4] re-calculated the experiments, and deduced the law,

$$\rho = 0\cdot030586\ (1 + \cdot0025\, v)\, v^2 \pi R^2 \text{ in French measures.}$$

Afterwards M. Didion[5] examined them and obtained the law,

$$\rho = 0\cdot027\,(1 + \cdot00257\, v)\, v^2 \pi R^2,$$

and M. Didion[6] gives as his final result, deduced from the Metz experiments,

$$\rho = 0\cdot027\,(1 + \cdot0023\, v)\, v^2 \pi R^2,$$

concluding his "Lois de la Résistance" with the remark on this formula: "Elle peut donc être appliquée avec confiance au tir de tous les projectiles *sphériques* en usage." It is quite plain that Hutton's experiments could not indicate any particular law with great precision since such varied ormulæ have been deduced from them.

The objections to this mode of experimenting are numerous and self-evident. As the weight of the ball is increased, the vibration and shock caused in the pendulum must give rise to large errors. With the increased distance, there is the difficulty of striking the pendulum so that there may be no impulse on the axis of suspension, and no tendency to twist about a

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[1] Hutton, Tracts, 1812.

[2] Didion, Traité de Balistique, 1848; 2nd Edition, 1861. Lois de la Résistance de l'Air, 1857.

[3] Hutton, Tract 37, p. 220.

[4] Didion, Traité, 1848, p. 44.

[5] Didion, Traité, 1861, p. 64.

[6] Didion, Traité, 1848, p. 52; 1861, p. 66; Lois, 1857, p. 79.

vertical axis. Further, as experiments have been commonly made, the shot were not removed from the receiver until a certain number of rounds had been fired, and thus the positions of the centre of gravity and centre of percussion were in a state of constant variation. There was also the uncertainty arising from the necessity for assuming that the *average* initial velocities were the same for the same charge and ball for different distances of the receiver. The large guns now made cannot be experimented on with the ballistic pendulum in the ordinary way.

I have been particular in referring to dates in what precedes, because M. Didion's *Traité de Balistique* is our standard work on the subject. The theory of the edition of 1861 is practically the same as that of 1848, depending, not only upon the same law of the resistance of the air, but upon the same numerical formula of resistance. The tables to facilitate the practical application of the theory are also, to all intents, the same. We must therefore date this theory 1848, before the Crimean war, and before the introduction of rifled ordnance and projectiles of a cylindrical form. It is true that M. Didion does make some slight reference to small elongated projectiles, diameter 0m.119 (4·7 in.) and length 0m.240 (9·4 in.) for he remarks, "On reviendra, section ix., sur les résistances de divers genres que l'air fait éprouver aux balles oblongues; mais *en attendant que des expériences plus précises aient fourni des résultats plus certains,* nous admettrons pour les balles oblongues pleines et pour les boulets oblongs, un coefficient de résistance égal aux deux tiers de celui de la balle sphérique, c'est-à-dire $A = 0{\cdot}081$ et $\rho = 0{\cdot}018\ (1 + 0{\cdot}0023v)$; il en sera les trois-quarts pour les balles creuses, comme celle du modèle 1859, c'est-à-dire égal à 0·020. On adoptera, pour les boulets oblongs de campagne et de siége, le même coefficient $A = 0{\cdot}018$."[1] It is plain, however, that the resistance of the air to elongated shot depends greatly upon the form of the shot, and that something far more precise than the above is required.

It is remarkable that Hutton concluded from his experiments that, for every 100 feet added to the velocity of a shot, there was an increasing addition made to the resistance of the air up to a velocity between 1600 f.s. and 1700 f.s., where it attained a maximum. Thus, in his table of the air's resistance to a ball of 2 inches in diameter, we find[2]

vel. f.s.	Resistance in oz.	Δ_1	Δ_2
1300	683·3		
		+128·2	
1400	811·5		+7·4
		+135·6	
1500	947·1		+4·2
		+139·8	
1600	1086·9		+1·7
		+141·5	
1700	1228·4		−1·3
		+140·2	
1800	1368·6		−3·1
		+137·1	
1900	1505·7		−5·0
		+132·1	
2000	1637·8		

[1] Didion, Traité, p. 74, 1861.

[2] Hutton, Tract 37, p. 218.

This asserted maximum is in itself very improbable, and M. Didion[1] declares that there is none deducible from Hutton's experiments. Hutton's attempt to account for this supposed maximum, by reference to the velocity with which air rushes into a vacuum, is extremely fanciful—because the after-pressure is not great to begin with, it diminishes rapidly as the velocity is increased, but perhaps never entirely vanishes.

Professor Wheatstone applied electricity to measure the velocities of shot in 1840. Afterwards, several changes were made in his instrument. Drawings were shewn in Paris, in May, 1841, and a copy was prepared for Colonel Konstantinoff; but he desired to have an instrument by which he could determine the *velocity of a shot at different points of its path.*[2] M. Breguet afterwards constructed an instrument for this purpose, the performance of which will be further noticed hereafter. This led to a dispute between Professor Wheatstone and M. Breguet, respecting their claims to the invention of the chronograph.[3] Pouillet attempted to determine short intervals of time on the supposition that the deflection of a magnet needle is proportional to the strength of a galvanic current, and to the time it is acted upon by the current. Hipp objected to Professor Wheatstone's arrangement, and made use of a clock going uniformly to measure his time. It does not appear that any of these chronographs have come into use, or that they have given results of any importance.

The electro-ballistic pendulum of Major Navez was invented about 1848, but was brought prominently into notice only in 1855. Very various opinions have been expressed respecting the value of this instrument. The Americans tried it and rejected it on account of the irregularity of its indications. The French used it in 1856, 1857, and 1858, at Metz, to complete their researches on the resistance of the air. "Les résultats n'ont pas été formulés, ils font voir qu'aux vitesses moyennes la valeur de ρ' est égale à celle qui a déjà été donnée, mais que pour des vitesses plus petites la résistance diminuerait *beaucoup plus rapidement* que par la formule déjà obtenue (Art. 52, éq. 7), et qu'elle décroîtrait même trop rapidement pour être dès maintenant admise".[4] Experiments with small arms were continued in 1857 and 1858, with new precautions, on which M. Didion observes: "Elles ont présenté, comme les précédentes, une diminution trop rapide dans les valeurs de ρ', et les résultats *n'ont pas donné une formule qu'on puisse appliquer au tir avec une entière confiance*"[4]. Others who have used the instrument have expressed a very high opinion of its accuracy. It is undoubtedly a great improvement upon all instruments that preceded it for rough practical work, but it is wanting in that precision we are entitled to ask for when electricity is brought to our assistance. It may well be doubted whether anything has been contributed to our knowledge of the laws of the resistance of the air by Major Navez's electro-ballistic pendulum, or by the modifications of it by De Brettes, Vignotti, Leurs, &c. In 1861, it is plain that M. Didion gave the preference to the results of the old ballistic pendulum.

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[1] Didion, Traité, p. 64, 1861.
[2] Müller, Berichte, p. 856, 1848.
[3] Moigno, Traité de télégraphie électrique,
[4] Didion, Traité, 1861, p. 71.