GENERAL METHOD OF SOLVING EQUATIONS OF ALL DEGREES

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GENERAL METHOD

OF

SOLVING EQUATIONS OF ALL DEGREES;

APPLIED PARTICULARLY TO

EQUATIONS OF THE SECOND, THIRD, FOURTH, AND FIFTH DEGREES.

BY

OLIVER BYRNE,

INVENTOR OF DUAL ARITHMETIC, A NEW ART; AND
THE CALCULUS OF FORM, A NEW SCIENCE.

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1868.

PREFACE.

THE chief business of the science of algebra is to evolve the value of unknown quantities from algebraical expressions termed *equations*, in which known and unknown quantities are involved, bearing to each other given relations which can be expressed numerically. It is well known that, to establish general formulæ by which the numerical values of the roots of equations of all degrees might be determined has baffled the most ardent exertions of the ablest analysts, and that the numerical methods employed to effect the same purpose are so laborious that their practical application is almost impossible. Notwithstanding, the general method of solving equations of all degrees, which I have established and illustrated in the following pages, is general, easily applied, and pre-eminently practical, and is, without doubt, the greatest acquisition that the science of algebra has received, for without such general method the science would be incomplete and lack one of its greatest requirements.

To be able to apply this method it is necessary to have a knowledge of the art and science of Dual Arithmetic which I invented and developed in five volumes lately published. Yet, for those who have not investigated Dual Arithmetic, it may be necessary to state, that any two of the three corresponding numbers (*Natural number*), (*Dual number*), (Dual Logarithm), may be almost instantly found, the remaining one being given; and that too, without the use of tables, by easy independent and direct processes. A dual number is written thus:—

$$\downarrow u_1, \ 'u_2 \ u_3, \ u_4, \ u_5, \ 'u_6 \ . \ . \ . \ . \ 'u_n, \quad (A);$$

u_1, u_3, u_4, u_5 are dual digits of the ascending branch in the 1st, 3rd, 4th, and 5th positions after the arrow, marked by numerals and commas to the right *below*, $'u_2$ $'u_6$ are dual digits of the descending branch in the 2nd and 6th positions after the arrow, marked by numerals and commas to the left *above*. This dual number represents the continued product of $(1{\cdot}1)^{u_1}$; $({\cdot}99)^{'u_2}$; $(1{\cdot}001)^{u_3}$; $(1{\cdot}0001)^{u_4}$; $(1{\cdot}00001)^{u_5}$; and $({\cdot}999999)^{'u_6}$. The consecutive bases of the ascending branch are, 1·1; 1·01; 1·001; 1·0001, &c.; and those of the descending ·9; ·99; ·999; ·9999, &c. The dual digits of any dual number (A), may be made to assume an immense number of values without altering the corresponding natural number; and each of such dual numbers, corresponding to the same natural number, may be reduced to a constant number in the *n*th position, leaving a zero in every other position; this constant number in the *n*th position is termed a dual logarithm of that position. The dual logarithm of the natural number a is written, $\downarrow, (a)$; $\downarrow, (y + \frac{1}{y})$ represents the dual logarithm of $y + \frac{1}{y}$; and so on; the comma being placed near the barb of the arrow.

In the same manner $\downarrow, u_1, u_2, 'u_3$ indicates the dual logarithm of the dual number $\downarrow u_1, u_2, 'u_3$. The accomplished mathematician must not consider my minute discussions of simple elementary propositions unnecessary, for it is my design that this method of solving equations of all degrees may be readily acquired by any student who understands the elements of algebra and common arithmetic.

OLIVER BYRNE.

GENERAL METHOD

OF

SOLVING EQUATIONS OF ALL DEGREES.

IN supposing w, in (1), to have a continuous range of numerical values, no new result is obtained by imagining v to be a proper fraction of the form $\frac{1}{z}$; for then (1) becomes (2).

$$v + \frac{1}{v} = w, \ (1);$$

$$z + \frac{1}{z} = w, \ (2).$$

Hence it is unnecessary to suppose v in (1) to have a value less than 1·, for the same resulting value of w may be obtained by giving to v its corresponding value greater than 1·. If w is negative then v and $\frac{1}{v}$ must be negative; and (3) becomes (4).

$$v + \frac{1}{v} = -w, \ (3);$$

$$-v - \frac{1}{v} = -w, \ (4).$$

Consequently the value of v in (3) is the same as the value of v in (1), numerically, but negative. And, consequently, no whole number or fraction, positive or negative, substituted for v in (1) will render $v + \frac{1}{v}$ numerically less than $+2$ or -2. Therefore, all such equations as $v + \frac{1}{v} = \pm w$, may be put under the form (1), in which, v may be always considered greater than unity, without interfering with the continuity of the numerical values of w. From (1) we obtain (5),

$$1 + \frac{1}{v^2} = w\frac{1}{v}, \quad (5).$$

Since v is a positive number greater than 1·, the left-hand member of (5) is always greater than 1· but less than 2·; whence the right-hand member of (5), that is, $w\frac{1}{v}$, must have a value existing between the same limits, but not beyond.

In supposing w in (6) to have a continuous range of numerical values, no new result is obtained in supposing v to be a proper fraction of the form $-\frac{1}{z}$; for then (6) becomes (7).

$$v - \frac{1}{v} = w, \quad (6);$$

$$-\frac{1}{z} + z = w, \quad (7).$$

It is therefore unnecessary to suppose v in (6) to have a value less than $(-1\cdot)$; for the same resulting value of w may be obtained by giving to v its corresponding positive value z greater than 1. Nor can v in (6) be a negative whole number, for then $v - \frac{1}{v}$ would become negative; and equal to w which is supposed to be positive. If $\frac{1}{z}$ be substituted for v in (6) it becomes $\frac{1}{z} - z = w$, or

$$z - \frac{1}{z} = -w, \quad (8).$$

Whence, if the value of v be known in (6), the value of z in (8) becomes known, for z in (8) is equal to $\frac{1}{v}$ in (6). v cannot be $= +1$ or -1 in (6), for then $v - \frac{1}{v} = 0$, which is absurd, for v is always = the whole number w; w being positive in (6), v must be a positive whole number, for if it be a proper fraction $+\frac{1}{z}$, it assumes the form (8), in which w is negative. And if v be a negative fraction $-\frac{1}{z}$, (6) assumes the identical form (7), in which z is a whole positive number.

Therefore all such equations as $v - \frac{1}{v} = \pm w$, may be solved by solving (6), in which v may be always considered greater than unity, without interfering with the continuity of the numerical values of $+ w$.

From (6) we obtain (9),

$$1 - \frac{1}{v^2} = w \frac{1}{v}, \ (9).$$

Since v in all cases is a positive number greater than 1, the left hand member of (9) is always greater than 0·, but less than 1·; whence the right-hand member of (9), that is $w \frac{1}{v}$, must have a value existing between the same limits 0· and 1·, but not beyond.

When w is not considered as standing for all possible numbers between its known limits, but has a particular value n, greater than 2·; that is,

$$v + \frac{1}{v} = n;$$

then if $a = v$ or $a + \frac{1}{a} = n$, also will $\frac{1}{a} = v$;

for $\frac{1}{a} + \frac{1}{\frac{1}{a}} = \frac{1}{a} + a = n$.

Again in $v - \frac{1}{v} = m$, then if $v = b$,

or $b - \frac{1}{b} = m$; v also $= - \frac{1}{b}$;

for, $- \frac{1}{b} - \left(\frac{1}{-\frac{1}{b}}\right) = - \frac{1}{b} + b = m$.

Lemma I.

If $\cdot f$ be put for any decimal fraction as ·34567888, and $f_1 = 3456788{\cdot}8$, $f_2 = 345678{\cdot}88$, $f_3 = 34567{\cdot}888$; &c.

Then, for dual numbers of not more than eight consecutive dual digits, we have

$$\downarrow,(1+\cdot f\downarrow u_{8,}) = \downarrow,(1+\cdot f)+\frac{f_8 u_8}{1+\cdot f}$$

$$\downarrow,(1+\cdot f\downarrow u_{7,}) = \downarrow,(1+\cdot f)+\frac{f_7 u_7}{1+\cdot f}$$

$$\downarrow,(1+\cdot f\downarrow u_{6,}) = \downarrow,(1+\cdot f)+\frac{f_6 u_6}{1+\cdot f}$$

$$\downarrow,(1+\cdot f\downarrow u_{5,}) = \downarrow,(1+\cdot f)+\frac{f_5 u_5}{1+\cdot f}$$

All of which are true to the required degree of accuracy. f_8=·34567888; f_7=3·4567888; f_6=34·567888; f_5=345·67888. $\downarrow, u_{1,}$ = 9531018, u_1; $\downarrow, u_{2,}$ = 995033, u_2; $\downarrow, u_{3,}$ = 99950, u_3; $\downarrow, u_{4,}$ = 10000, u_4; &c.

$$\downarrow,(1+\cdot f\downarrow' u_8) = \downarrow,(1+\cdot f)-\frac{f_8 u_8}{1+\cdot f}$$

$$\downarrow,(1+\cdot f\downarrow' u_7) = \downarrow,(1+\cdot f)-\frac{f_7 u_7}{1+\cdot f}$$

$$\downarrow,(1+\cdot f\downarrow' u_6) = \downarrow,(1+\cdot f)-\frac{f_6 u_6}{1+\cdot f}$$

$$\downarrow,(1+\cdot f\downarrow' u_5) = \downarrow,(1+\cdot f)-\frac{f_5 u_5}{1+\cdot f_8}$$

Within the designed degree of accuracy, which may be as great as we please, these equations hold exactly true, while the places of figures of the whole number f are not greater than half the places in $1+\cdot f$;

Example.

1+·50000000
16539
1+·50016539 Sum.
1+·49983461 Diff.

$$\downarrow,(1+\cdot 50000000)+\frac{16539}{1\cdot 5} = \downarrow,(1\cdot 50016539)$$

$$\downarrow,(1+\cdot 50000000)-\frac{16539}{1\cdot 5} = \downarrow,(1\cdot 49983461)$$

Proof.

$\downarrow$, (1·50016539) = 40557538, } 11028,
$\downarrow$, (1·50000000) = 40546512, } Differences
$\downarrow$, (1·49983461) = 40535484, } 11028,

$$\frac{16539}{1\cdot 5} = 11026.$$