

**SINGULAR PROPERTIES OF  
THE ELLIPSOID, AND  
ASSOCIATED SURFACES  
OF THE NTH DEGREE**

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Singular Properties of the Ellipsoid, and Associated Surfaces of the Nth Degree by G. F. Frederick Childe

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**G. F. FREDERICK CHILDE**

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SINGULAR PROPERTIES

OF THE

ELLIPSOID,

AND

ASSOCIATED SURFACES OF THE  $N^{\text{TH}}$  DEGREE.

DEDICATED, BY PERMISSION,

TO

HIS ROYAL HIGHNESS PRINCE ALFRED.

BY THE REV. G. F. CHILDE, M.A.,

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TO HIS ROYAL HIGHNESS  
FRINCH ALFRED,  
IN REMEMBRANCE OF HIS VISIT TO THE COLONY OF THE  
CAPE OF GOOD HOPE, IN THE YEAR 1860,  
AND OF HIS PERMISSION THEN GRACIOUSLY GIVEN,  
THIS VOLUME IS DEDICATED,  
BY HIS FAITHFUL, HUMBLE SERVANT,  
THE AUTHOR.





## INTRODUCTORY REMARKS.

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As the title of this volume indicates, its object is to develop peculiarities in the Ellipsoid; and, further, to establish analogous properties in the unlimited congeneric series of which this remarkable surface is a constituent.

The more conspicuous Singularities which have been evolved are enumerated in the Table of Contents; among them it seems desirable to specify more pointedly, and as briefly as possible, those which follow.

(1.)—The relation of the Circle-ordinate  $x$  to the co-ordinates. This relation is first noticed in (6), p. 2; and will be found to pervade the whole subsequent investigation. (2.)—The equation (13), p. 4; and the cognate relations developed throughout Chapter IV. (3.)—The series of equations exhibited in page 8. (4.)—The very peculiar relation of the intercept  $\rho_m$  to the co-ordinates, investigated for the Ellipsoid in (4), p. 10; and, for the General Surface, in (4), p. 22. 5.—The summary of General Formulæ commencing at p. 23. In the Table of Errata it is pointed out that the forms (5) (8) (9) are erroneous, and the requisite corrections have been there introduced. 6.—The remarkable system of equations representing successive planes in the ellipsoid, p. 30; and tangent-planes, p. 33. 7.—The developments of Chapter III. The geometrical peculiarities detected by means of these developments, pp. 51 and 56, deserve to be particularised. (8.)—The singular series of expressions for the tangent perpendicular in consecutive surfaces. (9.)—Properties of the Circle-ordinates in pp. 97 and 102. (10.)—The axes of the  $m^{\text{th}}$  derived ellipsoid, p. 109. (11.)—Singular relations involving the Circle-ordinates in consecutive surfaces, p. 113. (12.)—The singular relations combining consecutive co-ordinates and constants, in different surfaces, p. 125. (13.)—The relation uniting the Circle-ordinates in any three consecutive surfaces (XXXII), p. 130. (18.)—The singular equation connecting successive Circle-ordinates in the general surface, p. 133.

In Chapter VI, relating to the Ellipsoid, the area of a plane section, which is determined under the most general circumstances, has a singular symmetry of expression. It is possible that this result may be otherwise obtained in

some more compendious way, although no method more convenient has occurred to the author. The identification of the area with a second orthographical projection of the parallel central section, given at page 140, illustrates the clearness of interpretation which may attach to results of a purely analytical character, by reference to their geometrical equivalence. In applying the expression for a sectional area to the Volume of any portion of the solid limited by parallel planes, we are led to results of no ordinary simplicity regarding the volume or mass; while the extension of this investigation in determining the attractive force, under the hypothesis that the attraction varies directly with the distance, may not be without its value, in the consideration of problems relating to this subject.

With reference to this point, it should be remarked that, in the formula (2), page 148, the Density may be assumed to be any function of the distance, although, in the subsequent propositions, it has been taken as constant. That illustrations in greater detail, as well on this, as on some other topics, have not been given, must be attributed to the narrow limits within which the writer has been unavoidably restricted.

If it is of acknowledged difficulty in the present day, to bring forward anything of novelty in mathematical researches, yet it may be hoped that such attempts are not often altogether futile, when entered upon in the spirit which is anxious for the investigation of Truth, and desires to add its contribution to the treasury of Science.

To do this, in some degree, is the design of the investigations exhibited in the following pages. For whatever defect in its execution may be apparent, the reader's indulgence is requested; while, for the recognition of whatever truth has been elicited, the author is content to rest upon the candour of an impartial judgment.

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\* Note.—The formulæ (5) (8) (9), at pages 24 and 25, which are inaccurate in the text, have been corrected in the table of errata.