

**INTRODUCTION TO
QUATERNIONS, WITH
NUMEROUS EXAMPLES**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649615384

Introduction to Quaternions, with Numerous Examples by P. Kelland & P. G. Tait

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P. KELLAND & P. G. TAIT

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BY

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London:

MACMILLAN AND CO.

1873.

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181. f. 35

PREFACE.

THE present Treatise is, as the title-page indicates, the joint production of Prof. Tait and myself. The preface I write in the first person, as this enables me to offer some personal explanations.

For many years past I have been accustomed, no doubt very imperfectly, to introduce to my class the subject of Quaternions as part of elementary Algebra, more with the view of establishing principles than of applying processes. Experience has taught me that to induce a student to think for himself there is nothing so effectual as to lay before him the different stages of the development of a science in something like the historical order. And justice alike to the student and the subject forbade that I should stop short at that point where, more simply and more effectually than at any other, the intimate connexion between principles and processes is made manifest. Moreover in lecturing on the groundwork on which the mathematical sciences are based, I could not but bring before my class the names of great men who spoke in other tongues and belonged to other nationalities than their own—Diophantus, Des Cartes, Lagrange, for instance—and it was not just to omit the name of one as

great as any of them, Sir William Rowan Hamilton, who spoke their own tongue and claimed their own nationality. It is true the name of Hamilton has not had the impress of time to stamp it with the seal of immortality. And it must be admitted that a cautious policy which forbids to wander from the beaten paths, and encourages converse with the past rather than interference with the present, is the true policy of a teacher. But in the case before us, quite irrespective of the nationality of the inventor, there is ample ground for introducing this subject of Quaternions into an elementary course of mathematics. It belongs to first principles and is their crowning and completion. It brings those principles face to face with operations, and thus not only satisfies the student of the mutual dependence of the two, but tends to carry him back to a clear apprehension of what he had probably failed to appreciate in the subordinate sciences.

Besides, there is no branch of mathematics in which results of such wide variety are deduced by one uniform process; there is no territory like this to be attacked and subjugated by a single weapon. And what is of the utmost importance in an educational point of view, the reader of this subject does not require to encumber his memory with a host of conclusions already arrived at in order to advance. Every problem is more or less self-contained. This is my apology for the present treatise.

The work is, as I have said, the joint production of Prof. Tait and myself. The preface I have written without consulting my colleague, as I am thus enabled

to say what could not otherwise have been said, that mathematicians owe a lasting debt of gratitude to Prof. Tait for the singleness of purpose and the self-denying zeal with which he has worked out the designs of his friend Sir Wm. Hamilton, preferring always the claims of the science and of its founder to the assertion of his own power and originality in its development. For my own part I must confess that my knowledge of Quaternions is due exclusively to him. The first work of Sir Wm. Hamilton, *Lectures on Quaternions*, was very dimly and imperfectly understood by me and I dare say by others, until Prof. Tait published his papers on the subject in the *Messenger of Mathematics*. Then, and not till then, did the science in all its simplicity develop itself to me. Subsequently Prof. Tait has published a work of great value and originality, *An Elementary Treatise on Quaternions*.

The literature of the subject is completed in all but what relates to its physical applications, when I mention in addition Hamilton's second great work, *Elements of Quaternions*, a posthumous work so far as publication is concerned, but one of which the sheets had been corrected by the author, and which bears all the impress of his genius. But it is far from elementary, whatever its title may seem to imply; nor is the work of Prof. Tait altogether free from difficulties. Hamilton and Tait write for mathematicians, and they do well, but the time has come when it behoves some one to write for those who desire to become mathematicians. Friends and pupils have urged me to undertake this duty, and after consultation with Prof. Tait, who from

being my pupil in youth is my teacher in riper years, I have, in conjunction with him, and drawing unreservedly from his writings, endeavoured in the first nine chapters of this treatise to illustrate and enforce the principles of this beautiful science. The last chapter, which may be regarded as an introduction to the application of Quaternions to the region beyond that of pure geometry, is due to Prof. Tait alone. Sir W. Hamilton, on nearly the last completed page of his last work, indicated Prof. Tait as eminently fitted to carry on happily and usefully the applications, mathematical and physical, of Quaternions, and as likely to become in the science one of the chief successors of its inventor. With how great justice, the reader of this chapter and of Prof. Tait's other writings on the subject will judge.

PHILIP KELLAND.

UNIVERSITY OF EDINBURGH,
October, 1873.

CONTENTS.

CHAPTER I.

	PAGES
INTRODUCTORY	1-5

CHAPTER II.

VECTOR ADDITION AND SUBTRACTION	6-31
---	------

Definition of a vector, with conclusions immediately resulting therefrom, Art. 1-6; examples, 7; definition of UNIT VECTOR and TENSOR, with examples, 8; coplanarity of three coincident vectors, with conditions requisite for their terminating in a straight line, and examples, 9-13; mean point, 14.

ADDITIONAL EXAMPLES TO CHAPTER II.

CHAPTER III.

VECTOR MULTIPLICATION AND DIVISION	32-57
--	-------

Definition of multiplication, and first principles, Art. 15-18; fundamental theorems of multiplication, 19-23; examples, 23; definitions of DIVISION, VECTOR and QUATERNION, 24-28; examples, 29; conjugate quaternions, 30; interpretation of formulae, 31.

ADDITIONAL EXAMPLES TO CHAPTER III.