ELEMENTS OF PLANE AND SPHERICAL TRIGONOMETRY, WITH THE FIRST PRINCIPLES OF ANALYTIC GEOMETRY

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by James Thomson

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WITH THE

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OF

ANALYTIC GEOMETRY.

BY JAMES THOMSON, LL.D.

PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF GLASGOW.

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ADVERTISEMENT.

THE first edition of this work was intended chiefly as a Text-Book for the use of the students in the BELFAST INSTITUTION, when the Author was Professor of Mathematics in that establishment; and it was therefore written, not as a regular and complete treatise on Trigonometry, but as an outline to be filled up, and illustrated orally in the Lectures. It was received, however, by other readers in a more favourable manner than the Author could have anticipated from its nature and form; and, in consequence of this, he has been induced, in the subsequent editions, to make various alterations, and, it is hoped, improvements. The investigations, though still concise, are given at such length as to be easily understood by readers of ordinary talents and attainments; and various interesting additions are introduced, some from the best recent works on the subject, and others that have occurred to the Author himself. Of the latter kind are the improvements in the numerical resolution of triangles, established in Nos. 57, 58, 73, and 74, and exemplified in Nos. 62, 63, and 110; which, with the modes of operation previously known, seem to render the subject as simple and easy as can be desired. In the present edition, also, a scholium of considerable interest will be found at the end of the third section: and the last four pages of the ninth section contain some curious propositions in Spherical Geometry, most of which the Author believes to be new.

It has been everywhere the aim of the Author, to comprise in a small compass much useful and interesting matter; and, whatever may be the imperfections of the work, he trusts that the person who shall make himself well acquainted with what it contains, will find it easy to acquire a knowledge of all that is yet known in Trigonometry, and to apply it in Astronomy, and other branches of science.

In the present edition, the sines, tangents, &c. are defined as mere numbers or ratios. This mode of representing them has been in use for some time in the University of Cambridge; and it is attended with considerable advantage, particularly in the application of Trigonometry in Natural Philosophy. Should any persons prefer the common mode, they may have recourse to the Note at the end of the volume.

Glasgow College, July, 1844.

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ELEMENTS OF TRIGONOMETRY.*

I.—GENERAL PRINCIPLES.

1. EACH of the four parts into which a circle is divided by two diameters intersecting each other at right angles, is called a *quadrant*. If one of the four right angles be divided into 90 equal parts by radii of the circle, each of the parts is called a *degree*. The parts into which these radii divide the arc of the quadrant are (Euc. III. 27) all equal, and they are therefore called degrees of the circle. A sixtieth part of a degree is called a *minute*; and a sixtieth of a minute, a *second*.† Degrees, minutes, and seconds are denoted by the characters, $^{\circ}$, $'$, $''$.

2.* It is proved by writers on geometry, that the circumferences of different circles are proportional to their radii; and that, in the same circle, angles at the centre are proportional to the arcs on which they stand. Hence, if from the vertex of any angle A (*fig.* 1) as centre, two circumferences BCD and B′C′D′, be described, cutting one of the lines forming the angle in B and B′, and the other in C and C′, the ratio of the arc, BC, to its radius, AB, is equal to that of the other arc B′C′ to its radius AB′. This readily follows from the principles stated above: for, since, by No. 1, and Euc. I. 13, cor., all the angles about A amount to 360°, we have, by the second of those principles,

$$\frac{BC}{BCD}=\frac{A}{360^{\circ}},\ \text{and}\ \frac{B'C'}{B'C'D'}=\frac{A}{360^{\circ}};\ \text{whence}\ \frac{BC}{BCD}=\frac{B'C'}{B'C'D'}.$$

* TRIGONOMETRY, in its primitive meaning, is that branch of mathematical science, which determines certain sides or angles of a triangle from others that are known. It is of two kinds, *Plane* and *Spherical:* the former treating of triangles described on a plane; and the latter, of those on the surface of a sphere. The principles of trigonometry, however, are now of far more general application, furnishing means of investigation in almost every branch of mathematics.

† In some modern French works on mathematics, the *centesimal* division is adopted instead of the *sexagesimal;* the right angle, and consequently the quadrant, being divided into 100 degrees; the degree, into 100 minutes; and the minute, into 100 seconds. This division, however, is likely to fall into disuse.

Also, by the first principle, we have $\frac{BCD}{AB}=\frac{B'C'D'}{AB'}$.

Multiply the members of this equation by those of the preceding, and there will be obtained $\frac{BC}{AB}=\frac{B'C'}{AB'}$, which is the property above stated. Hence, if we assume any radius, and call it r, and if we denote the corresponding arc by s, $\frac{s}{r}$ will be always the same for the same angle, whatever may be the magnitude of r; and since s, and consequently, $\frac{s}{r}$, is proportional to the angle, whatever may be its magnitude, $\frac{s}{r}$ will be a correct measure of any angle whatever.*

3. If an angle be taken from a right angle, or an arc from a quadrant, the remainder is called the *complement* of that arc or angle. From this it follows, that if an angle or arc exceed 90°, its complement is negative.

4. If an angle be taken from two right angles, or an arc from a semicircle, the remainder is called the *supplement* of the angle or arc.

5. The straight line joining the extremities of an arc, is called its *chord*.

6. If from the vertex of any angle A (*fig.* 2, 4, 5, or 6) a circle be described with any radius, cutting the lines forming the angle in two points, B and C, and through one of these points C, a straight line be drawn perpendicular to the other line AB, and cutting it in D; the ratio of the perpendicular CD to the radius AC, that is, the number obtained by dividing the perpendicular by the radius, is called the *sine* of the angle A; and if DB be divided by the radius, the quotient is called the *versed sine* of the same angle. These, for the sake of abbreviation, are written sin A, and versin A, or vs A. Hence, calling the radius r, and multiplying by it, we get $CD = r \sin A$. The sine of an arc is evidently the ratio of half the chord of its double to the radius.

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* Hence, if s and r be equal, the angle becomes 1; and we thus see, that in this mode of measuring angles, the angle which is the unit, is that which has the circular arc on which it stands equal to the radius of the circle. Now, this angle is 57° 17′ 44″·8, or 206264″·8, nearly; as is found by the following analogy: 3·14159265 : 1 : : 180° : 57° 17′ 44″·8; the semicircumference of the circle whose radius is 1, being 3·14159265. (See the Author's Differential and Integral Calculus, page 41.) Hence, if the radius of a circular sector were 12 inches, and its arc 25 inches, we should find its angle to be 119° 21′ 58″·3, by multiplying 57°, &c. by 25, and dividing the product by 12.

7. If through the other point B another perpendicular to the same straight line, AB, be drawn, cutting the other line AC produced in E; the ratio of the perpendicular BE to the radius AB is called the *tangent* of the angle A; and the ratio of the hypotenuse AE to the radius is called the *secant* of the same angle. These, for abbreviation, are written tan A and sec A. Hence, by multiplying by the radius, we get $BE = r \tan A$, and $AE = r \sec A$; so that, by taking along with these the expression found at the end of the last No., we have the three formulas,

$$CD = r \sin A \ldots (a), \qquad BE = r \tan A \ldots (b), \qquad AE = r \sec A \ldots (c).$$

8. The *cosine* of an angle is the sine of its complement. In like manner, the *coversed sine, cotangent*, and *cosecant* of an angle are respectively the versed sine, tangent, and secant of its complement. Hence, since an angle is the complement of its complement, the cosine, cotangent, &c. of the complement of an angle are respectively its sine, tangent, &c. For brevity, the sine, versed sine, tangent, and secant of the complement of an angle A are written cos A, coversin A, or covs A, cot A, and cosec A.*

If now BA be produced to meet the circumference again in F, and the diameter GH be drawn perpendicular to BF, the angle CAG will (No. 3) be the complement of A. Drawing, therefore, CK and GL perpendicular to GH, and producing AC to meet GL in L, it follows from the last two Nos., that if CK, or its equal AD, be divided by the radius AC, the quotient will be the sine of CAG, or the cosine of A; and that, in like manner, if GK, GL, and AL be divided by the radius, the quotients will be covs A, cot A, and cosec A. Hence, by multiplying by the radius r (omitting the coversed sine, as giving a result of no value), we get the following formulas:

$$AD = r \cos A \ldots (d), \qquad GL = r \cot A \ldots (e), \qquad AL = r \operatorname{cosec} A \ldots (f).$$

9. If we regard the line AB as fixed, while AC, commencing its motion from coincidence with AB, revolves about the point A, the angle A will commence from nothing, and, by receiving continual increases, may attain any magnitude, however great. Thus, the revolving line (*fig.* 3) may take the successive positions AC_1, AC_2, AC_3, AC_4, AC_5, &c.; there being evidently no limit to the amount of angular space described by that line, which, like a crank in machi-

* This notation, and the corresponding ones in Nos. 6 and 7, possess great advantages from their conciseness, and from their suggesting to the mind at once the ideas which the symbols are intended to express.