

**A TEXT BOOK ON
THE METHOD OF
LEAST SQUARES**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649110377

A text book on the method of least squares by Mansfield Merriman

Except for use in any review, the reproduction or utilisation of this work in whole or in part in any form by any electronic, mechanical or other means, now known or hereafter invented, including xerography, photocopying and recording, or in any information storage or retrieval system, is forbidden without the permission of the publisher, Trieste Publishing Pty Ltd, PO Box 1576 Collingwood, Victoria 3066 Australia.

All rights reserved.

Edited by Trieste Publishing Pty Ltd.
Cover @ 2017

This book is sold subject to the condition that it shall not, by way of trade or otherwise, be lent, re-sold, hired out, or otherwise circulated without the publisher's prior consent in any form or binding or cover other than that in which it is published and without a similar condition including this condition being imposed on the subsequent purchaser.

www.triestepublishing.com

MANSFIELD MERRIMAN

**A TEXT BOOK ON
THE METHOD OF
LEAST SQUARES**

**WORKS OF
THE LATE MANSFIELD MERRIMAN**

PUBLISHED BY
JOHN WILEY & SONS, Inc.

MECHANICS OF MATERIALS, 64th Thousand, 5½ by 8½.
TREATISE ON HYDRAULICS, 50th Thousand, 5½ by 8½.
SANITARY ENGINEERING, 6th Thousand, 6 by 9.
METHOD OF LEAST SQUARES, 10th Thousand, 6 by 9.
ELEMENTS OF MECHANICS, 10th Thousand, 5 by 7½.
ELEMENTS OF HYDRAULICS, 8th Thousand, 5 by 7½.
MATHEMATICAL TABLES FOR CLASS USE, 5 by 7½.
SOLUTION OF EQUATIONS, 6 by 9.

REVISED BY
THADDEUS MERRIMAN

STRENGTH OF MATERIALS, 54th Thousand, 5 by 7½.

By **MANSFIELD MERRIMAN** and **HENRY S. JACOBY**
TEXT-BOOK ON ROOFS AND BRIDGES

PART I. STRESSES, 28th Thousand, 6 by 9.

PART II. GRAPHIC STATICS, 26th Thousand, 6 by 9.

PART III. BRIDGE DESIGN. *New Edition in Preparation.*

PART IV. HIGHER STRUCTURES, 11th Thousand, 6 by 9.

By **MANSFIELD MERRIMAN** and **JOHN P. BROOKS**
HANDBOOK FOR SURVEYORS, 10th Thousand, 4 by 6½.

MANSFIELD MERRIMAN, Editor-in-Chief

AMERICAN CIVIL ENGINEERS' HANDBOOK, 50th Thousand,
4½ by 7. Genuine leather or flexible binding.

Edited by
MANSFIELD MERRIMAN and **ROBERT S. WOODWARD**
MATHEMATICAL MONOGRAPHS, Twenty Volumes, 6
by 9.

A TEXT-BOOK OF
ON THE
METHOD OF LEAST SQUARES.

BY

MANSFIELD MERRIMAN,

LATE MEMBER OF AMERICAN MATHEMATICAL SOCIETY

EIGHTH EDITION, REVISED

TOTAL ISSUE, ELEVEN THOUSAND.

NEW YORK

JOHN WILEY & SONS, Inc.

LONDON: CHAPMAN & HALL, LIMITED

GA 275

M4

Astron. Dept.

Replacing 298 635

ASTRONOMY DEPT.

COPYRIGHT, 1884,
BY MANSFIELD MERRIMAN.

COPYRIGHT RENEWED, 1911,
BY MANSFIELD MERRIMAN.

Printed in U. S. A.

6/29

PRESS OF
BRAUNWORTH & CO., INC.
BOOK MANUFACTURERS
BROOKLYN, NEW YORK

PREFACE TO THE FIRST EDITION.

The "Elements of the Method of Least Squares," published in 1877, was written with two objects in view: first, to present the fundamental principles and processes of the subject in so plain a manner, and to illustrate their application by such simple and practical examples, as to render it accessible to civil engineers who have not had the benefit of extended mathematical training; and, secondly, to give an elementary exposition of the theory which would be adapted to the needs of a large and constantly increasing class of students.

In preparing the following pages the author has likewise kept these two objects continually in mind. While the former work has been used as a basis, the alterations and additions have been so numerous and radical as to render this a new and distinct book rather than a second edition. The arrangement of the theoretical and practical parts is entirely different. In Chapters I to IV is presented the mathematical development of the principles, methods, and formulas; while in Chapters V to IX the application of these

to the different classes of observations is made, and illustrated by numerous practical examples. For the use of both students and engineers, it is believed that this plan will prove more advantageous than the one previously followed. Hagen's deduction of the law of probability of error is given, as well as that of Gauss. More attention is paid to the laws of the propagation of error, the solution of normal equations, and the deduction of empirical formulas. Many new illustrative examples of the adjustment and comparison of observations have been selected from actual practice, and are discussed in detail. At the end of each chapter are given a few problems or queries; and in the Appendix are eight tables for abridging computations.

MANSFIELD MERRIMAN.

NOTE TO THE EIGHTH EDITION.

The seventh edition was the result of a thorough revision and was enlarged by the addition of new matter on the solution of normal equations, on the uncertainty of the probable error, and on the median. In this edition all known errors have been corrected and an alphabetical index has been added.

M. M.

CONTENTS.

CHAPTER I. ✓

INTRODUCTION.

CLASSIFICATION OF OBSERVATIONS	2
ERRORS OF OBSERVATIONS	3
PRINCIPLES OF PROBABILITY	6
PROBLEMS	12

CHAPTER II. ✓

LAW OF PROBABILITY OF ERROR.

AXIOMS DERIVED FROM EXPERIENCE	13
THE PROBABILITY CURVE	15
FIRST DEDUCTION OF THE LAW OF ERROR	17
SECOND DEDUCTION OF THE LAW OF ERROR	22
DISCUSSION OF THE PROBABILITY CURVE	25
THE PROBABILITY INTEGRAL	27
COMPARISON OF THEORY AND EXPERIENCE	31
REMARKS ON THE FUNDAMENTAL FORMULAS	33
PROBLEMS AND QUERIES	35

CHAPTER III. ✓

THE ADJUSTMENT OF OBSERVATIONS.

WEIGHTS OF OBSERVATIONS	36
THE PRINCIPLE OF LEAST SQUARES	38
DIRECT OBSERVATIONS ON A SINGLE QUANTITY	41
INDEPENDENT OBSERVATIONS OF EQUAL WEIGHT	43
INDEPENDENT OBSERVATIONS OF UNEQUAL WEIGHT	51
SOLUTION OF NORMAL EQUATIONS	56
CONDITIONED OBSERVATIONS	57
PROBLEMS	65

CHAPTER IV.

THE PRECISION OF OBSERVATIONS

THE PROBABLE ERROR	66
PROBABLE ERROR OF THE ARITHMETICAL MEAN	70
PROBABLE ERROR OF THE GENERAL MEAN	72
LAWS OF PROPAGATION OF ERROR	75
PROBABLE ERRORS FOR INDEPENDENT OBSERVATIONS	79
PROBABLE ERRORS FOR CONDITIONED OBSERVATIONS	86
PROBLEMS	87

CHAPTER V.

DIRECT OBSERVATIONS ON A SINGLE QUANTITY.

OBSERVATIONS OF EQUAL WEIGHT	88
SHORTER FORMULAS FOR PROBABLE ERROR	92
OBSERVATIONS OF UNEQUAL WEIGHT	95
PROBLEMS	99

CHAPTER VI.

FUNCTIONS OF OBSERVED QUANTITIES.

LINEAR MEASUREMENTS	101
ANGLE MEASUREMENTS	104
PRECISION OF AREAS	106
REMARKS AND PROBLEMS	107

CHAPTER VII.

INDEPENDENT OBSERVATIONS ON SEVERAL QUANTITIES.

METHOD OF PROCEDURE	109
DISCUSSION OF LEVEL LINES	110
ANGLES AT A STATION	117
EMPIRICAL CONSTANTS	124
EMPIRICAL FORMULAS	130
PROBLEMS	139