

**REPORT ON THE
QUANTUM THEORY
OF SPECTRA**

Published @ 2017 Trieste Publishing Pty Ltd

ISBN 9780649246373

Report on the Quantum Theory of Spectra by L. Silberstein

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BY
L. SILBERSTEIN, PH.D.

PUBLISHED BY
ADAM HILGER, LTD.
75A CAMDEN ROAD, LONDON, N.W. 1.

1920

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91-12-22. R.W.
Reclams 5-13-26 MAF

PREFACE

THE present report was written originally for the private use of Messrs. Adam Hilger, and was then, at their instance, amplified and prepared for publication. But even with this extension it will be found to contain a brief account of only the most important contributions to the Quantum Theory of Spectra, a new field of inquiry opened only six or seven years ago by Niels Bohr, but already very vast and rapidly growing and ramifying in multiple directions. It will, no doubt, soon call for a supplement. As it is, however, it is hoped to be a helpful guide for those desiring to enter upon this new and strangely fascinating line of thought and investigation.

I take the opportunity of expressing my best thanks to Messrs. Hilger for bringing me into closer contact with this refreshing new line of thought of the modern spectroscopists. My thanks are also due to my beloved teacher, Prof. Max Planck, and to my friend, Prof. A. W. Porter, for reading the proof slips, and to Messrs. MacLehose for the indefatigable care they have bestowed upon this book.

L. S.

RESEARCH DEPT. ADAM HILGER, LTD.,
LONDON, February 17, 1920.

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CONTENTS

	PAGE
1. BIBLIOGRAPHY	1
2. INTRODUCTORY REMARKS. BOHR'S ASSUMPTIONS AND FIRST RESULTS	2
3-4. RYDBERG'S CONSTANT AND FINITE MASS OF NUCLEUS	8
5. QUANTIZING PRINCIPLES. ELLIPTIC ORBITS AND POTENTIAL MULTIPLICITY OF SPECTRUM LINES	9
6. RESTRICTING INEQUALITIES. QUANTIZATION OF ORBITS IN SPACE	13
7. SOMMERFELD'S THEORY OF FINE-STRUCTURE. HY- DROGEN DOUBLETS AND HELIUM TRIPLETS	16
8. THEORY OF ROENTGEN SPECTRA	27
9. THEORY OF THE STARK-EFFECT	28
10. ZEEMAN-EFFECT. PHOTOELECTRIC EFFECT. PLANCK'S THEORY OF ROTATION SPECTRA. MISCELLANEOUS NOTES ON RECENT WORK	33
INDEX	41

REPORT ON THE QUANTUM THEORY OF SPECTRA.

1. The present report, which I shall attempt to bring up to date, is based upon the reading and scrutiny of the following original papers :

1. N. Bohr, *Phil. Mag.*, xxvi, pp. 1-25, 1913.
2. " " " " xxvi, 476-502, 1913.
3. " " " " xxvi, 857-875, 1913.
4. " " " " xxvii, 506-524, 1913.
- 4a. " " " " xxix, 332-335, 1915.
5. Th. Weyheide, *Annalen der Physik*, xlix, 966-1000, 1916.
- 5a. " " *ibid.*, lii, 276-290, 1917 (three notes).
6. M. Planck, *ibid.*, li, 385-418, 1916.
7. P. S. Epstein, *ibid.*, 489-520, 1916.
8. " " " " 815-840, 1916.
9. F. Paschen, *ibid.*, 901-940, 1916.
10. A. Sommerfeld, *Ann. d. Physik*, li, 1-94, 1916.
11. " " " " " " 125-167, 1916.
- 11a. K. Glitscher, *ibid.*, lii, 608-630, 1917.
12. P. S. Epstein, *ibid.*, li, 168-188, 1916.
13. K. F. Herzfeld, *ibid.*, 261-284, 1916.
14. P. Ehrenfest, *ibid.*, 327-352, 1916.
15. N. Bohr, *Danish Acad. Sc.*, IV, 1, Parts I and II, pp. 1-100, 1918.
16. M. Planck, *Annalen der Physik*, lii, pp. 491-505, and liii, pp. 241-256, 1917.
17. J. M. Burgers, *ibid.*, pp. 195-202, 1917.
18. H. A. Kramers, *Mémoires Acad. Sc., Copenhagen*, 8th ser., iii, No. 3, pp. 287-384, 1919.
19. L. Silberstein, *Phil. Mag.*, xxxix, January 1920.

These papers will, whenever the need occurs, be referred to by the attached numbers [in square brackets]. In addition to this

first-hand material may be mentioned Mr. J. H. Jeans' *Report on Radiation and the Quantum Theory*, Physical Society of London, 1914 (pp. iv+90), in which a few pages (pp. 50-57) are dedicated to Bohr's elementary theory of line spectra. Jeans' *Report* may be recommended for a rapid and easy initiation into Planck's theory of quanta in connection with black-body radiation, the knowledge of which I here presuppose, at least as far as its rudiments are concerned. With regard to the nucleus structure of the atoms (upon which all these spectrum theories are based) and to the experimental evidence for such a structure, it is advisable to consult Rutherford's papers, which are easily accessible (*Phil. Mag.**) and as easily read. There is no need for incorporating here a description of Rutherford's work.

2. The strong side of the quantum theory of spectra, as first proposed by Bohr [1] and further developed by Sommerfeld and others, consists in its very remarkable agreement with experiment, which in certain directions can be traced even to minute details of observations, such as the fine-structure of the 'lines' or groups of Fowler's helium series; the weak side of the theory consists in the heavy sacrifices it requires at the very outset, e.g. the abandonment of otherwise well-established principles of mechanics and of electromagnetism, in addition to the radical innovation inherent in the discontinuity of the very concept of Planck's quantum (quantum of 'action,' that is, 'Wirkungsquantum'), which is one of the most essential parts of the spectrum theory under consideration. The agreement, however, is of such a startling nature that the theory deserves, in spite of all these heavy sacrifices, and notwithstanding its somewhat magically arithmetical character, the greatest active interest of the modern physicist. Moreover, some of the recent predictions of the new theory require, for their experimental verification, a further refinement of spectroscopic apparatus and methods of observation, which soon may find an echo in the optician's workshop.

The best way to become acquainted with the fundamental assumptions, and with the chief results of the quantum theory of spectra will be to consider, in some detail, the simplest atomic

* The chief of these papers is published in vol. xxi of *Phil. Mag.* (1911), p. 669.

system, to wit, the hydrogen atom, which—according to Rutherford and his school—consists of a single electron (of mass m and charge $-e$) circling round a single positive *nucleus*, *i.e.* an electric positive charge ($+e$) contained in such a small volume that it represents almost entirely the mass of the hydrogen atom.*

In Bohr's first attempt at a spectrum theory [1, also 2, 3], the electrodynamic as well as the relativistic complications of the motion are disregarded, and only the purely electrostatic attraction between the nucleus and the electron is taken into account. In short, the motion is treated as ordinary Keplerian planetary motion (inverse square law), well-known since the beginnings of Celestial Mechanics. Moreover, to begin with, the mass of the nucleus is treated as infinite, in comparison with that of the planet or the electron, so that the vector equation of motion of the electron is simply

$$\ddot{\mathbf{r}} = -\frac{e^2}{m r^2} \mathbf{u}, \quad (1)$$

e being the charge in ordinary or so-called 'irrational,' electrostatic units, and $\mathbf{r} = r\mathbf{u}$, the vector drawn from the nucleus to the electron. It is scarcely necessary to say that writing down (1) as the equation of motion, the force-component corresponding to electromagnetic radiation is disregarded,—as already stated. But it is of importance to note that radiation is thus 'disregarded' not simply as a means of getting a first approximation, but it is disregarded radically, with purpose; it is *suspended*, in virtue of a postulate of the theory, precisely for the whole duration of validity of this smooth, Newtonian motion. In other words, apart from the small corrections due to relativity and due to the finite mass of the nucleus, the above equation (1) is assumed to hold *rigorously*, and the electron is assumed *not to radiate at all while it describes round the nucleus any of the Keplerian orbits* contained in (1).

And herein lies the chief of the sacrifices: the validity of Maxwellian principles is denied to these electronic orbits; notwithstanding that the motion of the electron is not uniform, it does not

* It will be kept in mind that 'mass' stands here for electromagnetic mass, and this is inversely proportional to the radius of the, say, spherical charge. Thus, according to Rutherford, the linear dimensions of the hydrogen nucleus would be about 1840 times smaller than those of an electron, and, therefore, of the order of 10^{-16} cm.