

**SUPERSONIC FLOW AND SHOCK  
WAVES: A MANUAL ON  
THE MATHEMATICAL THEORY  
OF NON-LINEAR WAVE MOTION**

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Supersonic Flow and Shock Waves: A Manual on the Mathematical Theory of Non-Linear Wave Motion by R. Courant

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**R. COURANT**

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# SUPERSONIC FLOW AND SHOCK WAVES

A MANUAL ON THE MATHEMATICAL THEORY  
OF NON-LINEAR WAVE MOTION

APPLIED MATHEMATICS PANEL  
NATIONAL DEFENSE RESEARCH COMMITTEE

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PREFACE

The following book differs from the usual reports issued under the auspices of the Office of Scientific Research and Development. It does not answer specific questions, and it does not contain summaries of results which a casual reader could use without studying the analytical details; instead, basic theoretical aspects of gas dynamics are presented in a rather mathematical form for the increasing number of well-trained scientists in related war work.

While trying to make practical contributions to problems of gas flow, Dr. K. C. Friedrichs and the undersigned have found a thorough understanding of the theoretical background indispensable; the present manual is an attempt to condense the result of their efforts in this theoretical direction. Before being drawn into work for the NDRC, the writers were preparing a set of lecture notes on topics of mathematical physics. This plan had to be postponed, but contact with classified publications and active participation in work on the theory of explosions and gas dynamics made it possible to write this manual, covering far more ground than planned originally for a chapter in a volume on wave propagation.

The content of the following pages is largely conditioned by personal experience and interest. No attempt at a balanced presentation was made. Even less was it possible to appraise the merits of all recent contributors to the field. The names of scientists with whom the authors had much contact will appear frequently, while others are hardly mentioned. This is also true of the bibliography which, incidentally, contains a few references to sources of general information for those readers who find that the present book starts at a point somewhat beyond their general background.

The book was written during, not after, a period of study and investigation while points of view underwent gradual changes. As a consequence, for example, scant attention is given to phenomena in liquids

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as compared with those in gases. Important subjects, such as flow of compressible fluids around obstacles, are hardly touched. Under conditions of more leisure such shortcomings should have been remedied.

A collective effort of the New York University Group of the Applied Mathematics Panel was necessary to produce the book. In particular, the authors are indebted to Messrs. Charles DePrima, Harvey Cohn and John Knudsen. Much labor is embodied in the drawings (by Mr. John Knudsen), most of which represent relevant actual cases. But Mr. Robert Shaw, more than anyone else, has rendered invaluable help in all matters of technical, literary and scientific character. Without his help the book would contain far more errors than it does now. Even so, it is full of imperfections; its publication in the present state was prompted by the feeling that further delay might make it useless for the purpose for which it was planned.

For obvious reasons, detailed references to classified material and specific applications are not provided, and thus the table of contents may convey an impression of overemphasis on theoretical aspects. The following comments may, therefore, be made.

Chapter I contains classical facts underlying any mathematical treatment of compressible flow.

Chapter II develops a theory of the type of partial differential equations which occur in treatable problems of compressible flow. An important point is the emphasis on what the authors call "simple waves", representing motion in domains adjacent to domains of constant state.

Chapter III is a rather extensive analysis of motion in one dimension. After an initial mathematical discussion the basic types of continuous motion, so-called rarefaction waves, are studied. Then follows an analysis of discontinuous motion, that is, of shock waves. The last part deals with the phenomena that occur when such elementary motions interact, e.g., when shock waves and rarefaction waves collide with, or overtake, each other. Ultimately every motion of a gas must be analyzed by a study of such interactions. The theory of detonation



waves, wave propagation in elastic-plastic solids under impact loading, and wave motion in open water channels are discussed briefly in the appendices to Chapter III.

Chapter IV deals with the case of steady two-dimensional flow, which presents itself most readily to a comprehensive mathematical analysis and, fortunately, provides an acceptable approximation to reality in many cases. Of particular interest to some readers may be the treatment of shock reflection, including the so-called Mach reflection.

Chapter V is of necessity the least systematic one. It deals with such problems in three dimensions as permit a reasonable theoretical attack. The first part concerns flow in nozzles and jets, a topic with increasingly important applications in many fields, e.g., rocket and jet propulsion. The second part is concerned with flow against conical obstacles such as projectiles, and gives an integrated summary of some work by Taylor and by Busemann. The problem of spherical waves, e.g., blast waves, is discussed very briefly in the last part of this chapter.

Altogether, the authors have tried to avoid discussions valuable mainly for their mathematical interest. Still, the book was written by mathematicians, and their willingness to accept compromise with an empirical approach does not make them physicists or engineers. Nevertheless, the authors hope that their effort will prove useful for the further development of the field.

R. Courant  
Technical Representative  
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