

A SHORT TABLE OF INTEGRALS

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A Short Table of Integrals by B. O. Peirce

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B. O. PEIRCE

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OF INTEGRALS**

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SHORT TABLE OF INTEGRALS.

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*The compiler will be grateful to any person who may send
notice of errors in these formulas to*

B. O. PEIRCE,
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I. FUNDAMENTAL FORMS.

1. $\int a dx = ax.$
2. $\int u f(x) dx = a \int f(x) dx.$
3. $\int \frac{dx}{x} = \log x.$
4. $\int x^m dx = \frac{x^{m+1}}{m+1},$ when m is different from $-1.$
5. $\int e^x dx = e^x.$
6. $\int a^x \log a dx = a^x.$
7. $\int \frac{dx}{1+x^2} = \tan^{-1} x.$
8. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x.$
9. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x.$
10. $\int \frac{dx}{\sqrt{2x-x^2}} = \text{versin}^{-1} x.$
11. $\int \cos x dx = \sin x.$
12. $\int \sin x dx = -\cos x.$

$$13. \int \operatorname{ctn} x \, dx = \log \sin x.$$

$$14. \int \tan x \, dx = -\log \cos x.$$

$$15. \int \tan x \sec x \, dx = \sec x.$$

$$16. \int \sec^2 x \, dx = \tan x.$$

$$17. \int \operatorname{csc}^2 x \, dx = -\operatorname{ctn} x.$$

In the following formulæ, u , v , w , and y represent any functions of x :

$$18. \int (u + v + w + \text{etc.}) \, dx = \int u \, dx + \int v \, dx + \int w \, dx + \text{etc.}$$

$$19a. \int u \, dv = uv - \int v \, du.$$

$$19b. \int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx.$$

$$20. \int f(y) \, dx = \int \frac{f(y) \, dy}{\frac{dy}{dx}}.$$

II. RATIONAL ALGEBRAIC FUNCTIONS.

A. — EXPRESSIONS INVOLVING $(a + bx)$.

The substitution of y or z for x , where $y = ax = a + bx$, gives

$$21. \int (a + bx)^n dx = \frac{1}{b} \int y^n dy.$$

$$22. \int x(a + bx)^n dx = \frac{1}{b^2} \int y^n (y - a) dy.$$

$$23. \int x^2(a + bx)^n dx = \frac{1}{b^{3+1}} \int y^n (y - a)^2 dy.$$

$$24. \int \frac{x^n dx}{(a + bx)^n} = \frac{1}{b^{n+1}} \int \frac{(y - a)^n dy}{y^n}.$$

$$25. \int \frac{dx}{x^n (a + bx)^n} = -\frac{1}{a^{n+1}} \int \frac{(z - b)^{n+n-2} dz}{z^n}.$$

Whence

$$26. \int \frac{dx}{a + bx} = \frac{1}{b} \log(a + bx).$$

$$27. \int \frac{dx}{(a + bx)^2} = -\frac{1}{b(a + bx)}.$$

$$28. \int \frac{dx}{(a + bx)^3} = -\frac{1}{2b(a + bx)^2}.$$

$$29. \int \frac{x dx}{a + bx} = \frac{1}{b^2} [a + bx - a \log(a + bx)].$$

$$30. \int \frac{x dx}{(a + bx)^2} = \frac{1}{b^2} \left[\log(a + bx) + \frac{a}{a + bx} \right].$$

$$81. \int \frac{x dx}{(a+bx)^2} = \frac{1}{b^2} \left[-\frac{1}{a+bx} + \frac{a}{2(a+bx)^2} \right]$$

$$82. \int \frac{x^2 dx}{a+bx} = \frac{1}{b^2} \left[\frac{1}{2}(a+bx)^2 - 2a(a+bx) + a^2 \log(a+bx) \right].$$

$$83. \int \frac{x^2 dx}{(a+bx)^2} = \frac{1}{b^2} \left[a+bx - 2a \log(a+bx) - \frac{a^2}{a+bx} \right].$$

$$84. \int \frac{dx}{x(a+bx)} = -\frac{1}{a} \log \frac{a+bx}{x}.$$

$$85. \int \frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)} - \frac{1}{a^2} \log \frac{a+bx}{x}.$$

$$86. \int \frac{dx}{x^2(a+bx)} = -\frac{1}{ax} + \frac{b}{a^2} \log \frac{a+bx}{x}.$$

B. — EXPRESSIONS INVOLVING $(a+bx^2)$.

$$87. \int \frac{dx}{c^2+x^2} = \frac{1}{c} \tan^{-1} \frac{x}{c}.$$

$$88. \int \frac{dx}{c^2-x^2} = \frac{1}{2c} \log \frac{c+x}{c-x}.$$

$$89. \int \frac{dx}{a+bx^2} = \frac{1}{\sqrt{ab}} \tan^{-1} x \sqrt{\frac{b}{a}}, \text{ if } a > 0, b > 0.$$

$$40. \int \frac{dx}{a+bx^2} = \frac{1}{2\sqrt{-ab}} \log \frac{\sqrt{a}+x\sqrt{-b}}{\sqrt{a}-x\sqrt{-b}}, \text{ if } a > 0, b < 0.$$

$$41. \int \frac{dx}{(a+bx^2)^2} = \frac{x}{2a(a+bx^2)} + \frac{1}{2a} \int \frac{dx}{a+bx^2}.$$

$$42. \int \frac{dx}{(a+bx^2)^{m+1}} = \frac{1}{2ma} \frac{x}{(a+bx^2)^m} + \frac{2m-1}{2ma} \int \frac{dx}{(a+bx^2)^m}.$$

$$43. \int \frac{x dx}{a+bx^2} = \frac{1}{2b} \log \left(x^2 + \frac{a}{b} \right).$$

$$44. \int \frac{x dx}{(a + bx^2)^{n+1}} = \frac{1}{2} \int \frac{dz}{(a + bz)^{n+1}}, \text{ where } z = x^2.$$

$$45. \int \frac{dx}{x(a + bx^2)} = \frac{1}{2a} \log \frac{x^2}{a + bx^2}.$$

$$46. \int \frac{x^2 dx}{a + bx^2} = \frac{x}{b} - \frac{a}{b} \int \frac{dx}{a + bx^2}.$$

$$47. \int \frac{dx}{x^2(a + bx^2)} = -\frac{1}{ax} - \frac{b}{a} \int \frac{dx}{a + bx^2}.$$

$$48. \int \frac{x^2 dx}{(a + bx^2)^{n+1}} = \frac{-x}{2mb(a + bx^2)^n} + \frac{1}{2mb} \int \frac{dx}{(a + bx^2)^n}.$$

$$49. \int \frac{dx}{x^2(a + bx^2)^{n+1}} = \frac{1}{a} \int \frac{dx}{x^2(a + bx^2)^n} - \frac{b}{a} \int \frac{dx}{(a + bx^2)^{n+1}}.$$

$$50. \int \frac{dx}{a + bx^2} = \frac{k}{3a} \left[\frac{1}{2} \log \left(\frac{(k+x)^2}{k^2 - kx + x^2} \right) + \sqrt{3} \tan^{-1} \frac{2x - k}{k\sqrt{3}} \right],$$

where $bk^2 = a$.

$$51. \int \frac{x dx}{a + bx^2} = \frac{1}{3bk} \left[\frac{1}{2} \log \left(\frac{k^2 - kx + x^2}{(k+x)^2} \right) + \sqrt{3} \tan^{-1} \frac{2x - k}{k\sqrt{3}} \right],$$

where $bk^2 = a$.

$$52. \int \frac{dx}{x(a + bx^n)} = \frac{1}{an} \log \frac{x^n}{a + bx^n}.$$

$$53. \int \frac{dx}{(a + bx^n)^{n+1}} = \frac{1}{a} \int \frac{dx}{(a + bx^n)^n} - \frac{b}{a} \int \frac{x^n dx}{(a + bx^n)^{n+1}}.$$

$$54. \int \frac{x^n dx}{(a + bx^n)^{p+1}} = \frac{1}{b} \int \frac{x^{n-p}}{(a + bx^n)^p} - \frac{a}{b} \int \frac{x^{n-n} dx}{(a + bx^n)^{p+1}}.$$

$$55. \int \frac{dx}{x^n(a + bx^n)^{p+1}} = \frac{1}{a} \int \frac{dx}{x^n(a + bx^n)^p} - \frac{b}{a} \int \frac{dx}{x^{n-n}(a + bx^n)^{p+1}}.$$