A SHORT TABLE OF INTEGRALS

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A Short Table of Integrals by B. O. Peirce

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B. O. PEIRCE

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Trieste

A

SHORT TABLE OF INTEGRALS.

COMPILED BY

B. O. PEIRCE,

HOLLIS PROFESSOR OF MATHEMATICS AND NATURAL PHILOSOPHY IN HARVARD UNIVERSITY.

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The compiler will be grateful to any person who may send notice of errors in these formulas to

B. O. PEIRCE,
Harvard College, Cambridge.

I. FUNDAMENTAL FORMS.

1. $\int a\,dx = ax.$

2. $\int af(x)\,dx = a\int f(x)\,dx.$

3. $\int \frac{dx}{x} = \log x.$

4. $\int x^m\,dx = \frac{x^{m+1}}{m+1}$, when m is different from -1.

5. $\int e^x\,dx = e^x.$

6. $\int a^x \log a\,dx = a^x.$

7. $\int \frac{dx}{1+x^2} = \tan^{-1}x.$

8. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x.$

9. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x.$

10. $\int \frac{dx}{\sqrt{2x-x^2}} = \text{versin}^{-1}x.$

11. $\int \cos x\,dx = \sin x.$

12. $\int \sin x\,dx = -\cos x.$

13. $\int \operatorname{ctn} x\,dx = \log \sin x.$

14. $\int \tan x\,dx = -\log \cos x.$

15. $\int \tan x \sec x\,dx = \sec x.$

16. $\int \sec^2 x\,dx = \tan x.$

17. $\int \csc^2 x\,dx = -\operatorname{ctn} x.$

In the following formulas, u, v, w, and y represent any functions of x:

18. $\int (u+v+w+\text{etc.})\,dx = \int u\,dx + \int v\,dx + \int w\,dx + \text{etc.}$

19a. $\int u\,dv = uv - \int v\,du.$

19b. $\int u \frac{dv}{dx}\,dx = uv - \int v \frac{du}{dx}\,dx.$

20. $\int f(y)\,dx = \int \frac{f(y)\,dy}{\frac{dy}{dx}}.$

II. RATIONAL ALGEBRAIC FUNCTIONS.

A. — Expressions Involving $(a+bx)$.

The substitution of y or z for x, where $y = xz = a + bx$, gives

21. $\int (a+bx)^n dx = \frac{1}{b}\int y^n dy.$

22. $\int x(a+bx)^n dx = \frac{1}{b^2}\int y^n (y-a) dy.$

23. $\int x^n (a+bx)^n dx = \frac{1}{b^{n+1}}\int y^n (y-a)^n dy.$

24. $\int \frac{x^n dx}{(a+bx)^m} = \frac{1}{b^{n+1}}\int \frac{(y-a)^n dy}{y^m}.$

25. $\int \frac{dx}{x^n (a+bx)^m} = -\frac{1}{a^{m+n-1}}\int \frac{(z-b)^{m+n-2} dz}{z^m}.$

Whence

26. $\int \frac{dx}{a+bx} = \frac{1}{b}\log(a+bx).$

27. $\int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)}.$

28. $\int \frac{dx}{(a+bx)^3} = -\frac{1}{2b(a+bx)^2}.$

29. $\int \frac{x\,dx}{a+bx} = \frac{1}{b^2}[a+bx-a\log(a+bx)].$

30. $\int \frac{x\,dx}{(a+bx)^2} = \frac{1}{b^2}\left[\log(a+bx)+\frac{a}{a+bx}\right].$

31. $\int \frac{x\,dx}{(a+bx)^3} = \frac{1}{b^2}\left[-\frac{1}{a+bx} + \frac{a}{2(a+bx)^2}\right]$.

32. $\int \frac{x^2\,dx}{a+bx} = \frac{1}{b^3}\left[\frac{1}{2}(a+bx)^2 - 2a(a+bx) + a^2\log(a+bx)\right]$.

33. $\int \frac{x^2\,dx}{(a+bx)^2} = \frac{1}{b^3}\left[a+bx - 2a\log(a+bx) - \frac{a^2}{a+bx}\right]$.

34. $\int \frac{dx}{x(a+bx)} = -\frac{1}{a}\log\frac{a+bx}{x}$.

35. $\int \frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)} - \frac{1}{a^2}\log\frac{a+bx}{x}$.

36. $\int \frac{dx}{x^2(a+bx)} = -\frac{1}{ax} + \frac{b}{a^2}\log\frac{a+bx}{x}$.

B. — Expressions Involving $(a+bx^n)$.

37. $\int \frac{dx}{c^2+x^2} = \frac{1}{c}\tan^{-1}\frac{x}{c}$.

38. $\int \frac{dx}{c^2-x^2} = \frac{1}{2c}\log\frac{c+x}{c-x}$.

39. $\int \frac{dx}{a+bx^2} = \frac{1}{\sqrt{ab}}\tan^{-1}x\sqrt{\frac{b}{a}}$, if $a>0,\ b>0$.

40. $\int \frac{dx}{a+bx^2} = \frac{1}{2\sqrt{-ab}}\log\frac{\sqrt{a}+x\sqrt{-b}}{\sqrt{a}-x\sqrt{-b}}$, if $a>0,\ b<0$.

41. $\int \frac{dx}{(a+bx^2)^2} = \frac{x}{2a(a+bx^2)} + \frac{1}{2a}\int \frac{dx}{a+bx^2}$.

42. $\int \frac{dx}{(a+bx^2)^{m+1}} = \frac{1}{2ma}\,\frac{x}{(a+bx^2)^m} + \frac{2m-1}{2ma}\int \frac{dx}{(a+bx^2)^m}$.

43. $\int \frac{x\,dx}{a+bx^2} = \frac{1}{2b}\log\left(x^2+\frac{a}{b}\right)$.

44. $\int \frac{x\,dx}{(a+bx^2)^{m+1}} = \frac{1}{2}\int \frac{dz}{(a+bz)^{m+1}}$, where $z = x^2$.

45. $\int \frac{dx}{x(a+bx^2)} = \frac{1}{2a}\log\frac{x^2}{a+bx^2}$.

46. $\int \frac{x^2\,dx}{a+bx^2} = \frac{x}{b} - \frac{a}{b}\int \frac{dx}{a+bx^2}$.

47. $\int \frac{dx}{x^2(a+bx^2)} = -\frac{1}{ax} - \frac{b}{a}\int \frac{dx}{a+bx^2}$.

48. $\int \frac{x^2\,dx}{(a+bx^2)^{m+1}} = \frac{-x}{2mb(a+bx^2)^m} + \frac{1}{2mb}\int \frac{dx}{(a+bx^2)^m}$.

49. $\int \frac{dx}{x^2(a+bx^2)^{m+1}} = \frac{1}{a}\int \frac{dx}{x^2(a+bx^2)^m} - \frac{b}{a}\int \frac{dx}{(a+bx^2)^{m+1}}$.

50. $\int \frac{dx}{a+bx^3} = \frac{k}{3a}\left[\frac{1}{2}\log\left(\frac{(k+x)^3}{k^3-kx+x^3}\right) + \sqrt{3}\tan^{-1}\frac{2x-k}{k\sqrt{3}}\right]$,

where $bk^3 = a$.

51. $\int \frac{x\,dx}{a+bx^3} = \frac{1}{3bk}\left[\frac{1}{2}\log\left(\frac{k^2-kx+x^2}{(k+x)^2}\right) + \sqrt{3}\tan^{-1}\frac{2x-k}{k\sqrt{3}}\right]$,

where $bk^3 = a$.

52. $\int \frac{dx}{x(a+bx^n)} = \frac{1}{an}\log\frac{x^n}{a+bx^n}$.

53. $\int \frac{dx}{(a+bx^n)^{m+1}} = \frac{1}{a}\int \frac{dx}{(a+bx^n)^m} - \frac{b}{a}\int \frac{x^n\,dx}{(a+bx^n)^{m+1}}$.

54. $\int \frac{x^m\,dx}{(a+bx^n)^{p+1}} = \frac{1}{b}\int \frac{x^{m-n}}{(a+bx^n)^p} - \frac{a}{b}\int \frac{x^{m-n}\,dx}{(a+bx^n)^{p+1}}$.

55. $\int \frac{dx}{x^m(a+bx^n)^{p+1}} = \frac{1}{a}\int \frac{dx}{x^m(a+bx^n)^p} - \frac{b}{a}\int \frac{dx}{x^{m-n}(a+bx^n)^{p+1}}$.