PLANE TRIGONOMETRY FOR THE USE OF COLLEGES AND SCHOOLS: WITH NUMEROUS EXAMPLES

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Plane Trigonometry for the Use of Colleges and Schools: With Numerous Examples by I. Todhunter

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I. TODHUNTER

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WITH NUMEROUS EXAMPLES.

BY

I. TODHUNTER, M.A., F.R.S.

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PREFACE.

THE present work contains all the propositions which are usually included in treatises on Plane Trigonometry, together with more than six hundred examples for exercise. The design has been to render the subject intelligible to beginners, and at the same time to afford the student the opportunity of obtaining all the information which he will require on this branch of Mathematics. The work is divided into a large number of chapters, each of which is in a great measure complete in itself. Thus it will be easy for teachers to select for pupils such portions as will be suitable for them in their first reading of the book. Each chapter is followed by a set of examples; those which are entitled *Miscellaneous Examples*, together with a few in some of the other sets, may be advantageously reserved by the student for exercise after he has made some progress in the subject.

As the text and the examples of the present work have been tested by considerable experience in teaching, the hope may be entertained that they will be suitable for imparting a sound and comprehensive knowledge of Plane Trigonometry, together with readiness in the application of this knowledge to the solution of problems. Any suggestions or corrections from students and teachers will be most thankfully received.

I. TODHUNTER.

ST. JOHN'S COLLEGE,
Feb. 21, 1859.

In the second edition the work has been revised, and the hints for the solution of the examples have been considerably increased.

December, 1860.

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PLANE TRIGONOMETRY.

I. MEASUREMENT OF ANGLES BY DEGREES OR GRADES.

1. The word Trigonometry is derived from two Greek words, one signifying a *triangle* and the other signifying *I measure*, and originally denoted the science in which the relations subsisting between the sides and angles of a triangle were investigated; the science was called *plane* trigonometry, or *spherical* trigonometry, according as the triangle was formed on a *plane* surface or on a *spherical* surface. Plane Trigonometry has now a wider meaning, and comprises all algebraical investigations with respect to plane angles, whether forming a triangle or not.

2. We have first to explain how angles are measured. A plane rectilineal angle is defined by Euclid as the inclination of two straight lines to one another which meet together, but are not in the same straight line. And when a straight line standing on another makes the adjacent angles equal to one another, each of the angles is called a *right angle*. A right angle is divided into 90 equal parts called *degrees*, a degree is divided into 60 equal parts called *minutes*, and a minute into 60 equal parts called *seconds*. Thus any angle may be estimated by ascertaining the number of degrees it contains; if the angle does not contain an exact number of degrees, we can express it in degrees and a fraction of a degree; or the fraction of a degree may be converted into minutes and seconds.

3. Thus, for example, half a right angle contains 45 degrees; a quarter of a right angle contains $22\frac{1}{2}$ degrees, which we may write

in the decimal notation 22·5 degrees, or we may express it as 22 degrees, 30 minutes. Similarly, if a right angle be divided into 16 equal parts, each part contains 5§ degrees, that is, 5 degrees, 37 minutes, 30 seconds.

4. Symbols are used as abbreviations of the words *degrees, minutes, seconds.* Thus 5° 37′ 30″ is used to denote 5 degrees, 37 minutes, 30 seconds.

5. The method of estimating angles by degrees, minutes, and seconds, is almost universally adopted in practical calculations. Another method was proposed in France in connexion with a uniform system of decimal tables of weights and measures. In this method a right angle is divided into 100 equal parts called *grades*, a grade is divided into 100 equal parts called *minutes*, and a minute is divided into 100 equal parts called *seconds*. On account of the occurrence of the number *one hundred* in forming the subdivisions of a right angle, this method of estimating angles is called the *centesimal* method; and the common method is called the *sexagesimal* method on account of the occurrence of the number *sixty* in forming the subdivisions of a degree. The centesimal method is also called the *French* method, and the common method is called the *English* method.

6. Symbols are used as abbreviations of the words *grades, minutes,* and *seconds*, in the centesimal method. Thus 5ᵍ 37ᶜ 30ˢˢ is used to denote 5 grades, 37 minutes, 30 seconds in the centesimal method. A centesimal minute and second are not the same as a sexagesimal minute and second, and the accents which are used to denote centesimal minutes and seconds differ from those which are used to denote sexagesimal minutes and seconds.

7. In the centesimal method any whole number of minutes and seconds may be expressed immediately as a decimal fraction of a grade. Thus 37 minutes is $\frac{37}{100}$ of a grade, that is ·37 of a grade; and 30 seconds is $\frac{30}{(100)^2}$ of a grade, that is ·003 of a grade.